## CIS 570 — Introduction to Formal Language Theory — Fall 2008

# Exercise Set 2

### Due by 4:00 p.m. on Tuesday, October 14

**Exercise 1 (20 points)**

Let $X = \{\, w \in \{\mathsf{0},\mathsf{1}\}^* \mid |w| \leq 4$ and $|w|$ is even and $\textbf{alphabet}\, w = \{\mathsf{0},\mathsf{1}\}\,\}$. Use Forlan to find and show the correctness of a regular expression $\alpha$ such that $L(\alpha) = X$. Try to minimize the size of $\alpha$, and use Forlan to display the size of $\alpha$. Try to do as much as possible of the work of finding and showing the correctness of $\alpha$ using Forlan. (Include a listing of your Forlan session.)

**Exercise 2 (30 points)**

**(a)** Prove that, for all $A, B \in \textbf{Lan}$, $(A^*B)^*A^* = (A \cup B)^*$. [25 points]

**(b)** Use Part (a) to prove that Simplification Rule (2) is correct: for all $\alpha, \beta \in \textbf{Reg}$, $(\alpha^*\beta)^*\alpha^* \approx (\alpha + \beta)^*$. [5 points]

**Exercise 3 (50 points)**

Define a function $\textbf{diff} \in \{\mathsf{0},\mathsf{1}\}^* \to \mathbb{Z}$ by: for all $w \in \{\mathsf{0},\mathsf{1}\}^*$,

$$\textbf{diff}\, w = \text{the number of } \mathsf{1}\text{'s in } w - \text{the number of } \mathsf{0}\text{'s in } w.$$

Thus:

- $\textbf{diff}\, \% = 0$;
- $\textbf{diff}\, \mathsf{0} = -1$;
- $\textbf{diff}\, \mathsf{1} = 1$;
- for all $x, y \in \{\mathsf{0},\mathsf{1}\}^*$, $\textbf{diff}(xy) = \textbf{diff}\, x + \textbf{diff}\, y$.

Let $X = \{\, w \in \{\mathsf{0},\mathsf{1}\}^* \mid \textbf{diff}\, w = 3m, \text{ for some } m \in \mathbb{Z}\,\}$.

**(a)** Find a regular expression $\alpha$ such that $L(\alpha) = X$. [15 points]

**(b)** Prove that your answer to Part (a) is correct. [35 points]