CIS 570 — Introduction to Formal Language Theory — Fall 2008

Final Examination

Thursday, December 18, 11:50 a.m.–1:40 p.m.

Question 1 (25 points)

(a) Let

$$X = \{\, \mathsf{0}^n \mathsf{1}^{2n} \mid n \in \mathbb{N} - \{0\} \,\} \qquad \text{and} \qquad Y = \{\, \mathsf{2}^{3n} \mathsf{3}^{4n} \mid n \in \mathbb{N} \,\}.$$

Find a grammar G such that $L(G) = (XY)^*$. [20 points]

(b) Draw one or more parse trees pt_1, pt_2, $\ldots$, pt_n such that:

- for all $i \in [1 : n]$, pt_i is valid for G, **rootLabel** $pt_i = s_G$ and **yield** $pt_i \in \{\mathsf{0}, \mathsf{1}, \mathsf{2}, \mathsf{3}\}^*$; and
- each production of G is used by at least one of the parse trees pt_1, pt_2, $\ldots$, pt_n.

Say what **yield** pt_1, **yield** pt_2, $\ldots$, **yield** pt_n are. [5 points]

Question 2 (30 points)

(a) Let

$$X = \{\, w \in \{\mathsf{0}, \mathsf{1}\}^* \mid |w| \text{ is even and } \mathsf{000} \text{ and } \mathsf{111} \text{ are substrings of } w \,\}.$$

Carefully explain how you could use Forlan to find a DFA M, with as few states as possible, such that $L(M) = X$. You should try to make Forlan do as much of the work of finding M as possible. [15 points]

(b) Let

$$Y = \{\, w \in X \mid \text{ there is no proper substring } v \text{ of } w \text{ such that } v \in X \,\}.$$

Carefully explain how you could use Forlan to turn the M of Part (a) into a DFA N, with as few states as possible, such that $L(N) = Y$. You should try to make Forlan do as much of the work of finding N as possible. [15 points]

Question 3 (45 points)

Define $\mathsf{blocks} \in \{\mathsf{0},\mathsf{1}\}^* \to \mathbb{N}$ by recursion:

- $\mathsf{blocks}\,\% = 0$;
- for all $w \in \{\mathsf{0},\mathsf{1}\}^*$,

$$\mathsf{blocks}(w\mathsf{0}) = \begin{cases} 1, & \text{if } w = \%, \\ \mathsf{blocks}\,w, & \text{if } \mathsf{0} \text{ is a suffix of } w, \\ \mathsf{blocks}\,w + 1, & \text{if } \mathsf{1} \text{ is a suffix of } w; \end{cases}$$

- for all $w \in \{\mathsf{0},\mathsf{1}\}^*$,

$$\mathsf{blocks}(w\mathsf{1}) = \begin{cases} 1, & \text{if } w = \%, \\ \mathsf{blocks}\,w, & \text{if } \mathsf{1} \text{ is a suffix of } w, \\ \mathsf{blocks}\,w + 1, & \text{if } \mathsf{0} \text{ is a suffix of } w. \end{cases}$$

For example, $\mathsf{blocks}\,\mathsf{001011110} = 5$, because $\mathsf{001011110} = (\mathsf{00})(\mathsf{1})(\mathsf{0})(\mathsf{1111})(\mathsf{0})$ is made up of 5 "blocks" of 0's and 1's: 00, 1, 0, 1111 and 0.

Let $X = \{\, w \in \{\mathsf{0},\mathsf{1}\}^* \mid \mathsf{blocks}\,w \text{ is even} \,\}$.

(a) Find a DFA M such that $L(M) = X$. [15 points]

(b) Prove that your answer to Part (a) is correct, using our standard technique for showing the correctness of DFAs, and making explicit use of the definition of the blocks function. [30 points]