Exercise 1 (45 points)

Let $X = \{ w \in \{0, 1, 2\}^* \mid \text{for all substrings } x \text{ of } w, \text{ if } |x| = 2, \text{ then } x \in \{01, 12, 20\} \}.$

(a) Find a regular expression $\alpha$ such that $L(\alpha) = X.$ [15 points]

(b) Prove that your answer to (a) is correct. [30 points]

Exercise 2 (55 points)

Let $X = \{ w \in \{0, 1\}^* \mid \text{for all } x, y \in \{0, 1\}^*, \text{ if } w = x00y, \text{ then } 11 \text{ is a substring of } y \}.$

(a) Find an FA $M$ such that $L(M) = X$ and $M$ has as few states as possible, i.e., for all FAs $N$, if $L(N) = X$, then $|Q_N| \geq |Q_M|.$ [15 points]

(b) Use Forlan to provide some evidence that $L(M) = X$, making use of some test cases that are in $X$, as well as some that are not in $X$. For the positive test cases, generate and display minimum-length labeled paths showing why the strings are accepted by $M$. (Include a transcript of your Forlan session.) [10 points]

(c) Prove that your answer to (a) is correct. [30 points]