Exercise 1 (20 points)

Let $M$ be the DFA

![DFA Diagram]

Give a step-by-step explanation of how our DFA minimization algorithm turns $M$ into a DFA $N$ with as few states as possible. Draw $N$ and use Forlan to check that your final answer is correct. (Include a transcript of your Forlan session.)

Exercise 2 (20 points)

Let $M$ be the FA

![FA Diagram]

Give a step-by-step explanation of how our FA-to-regular expression conversion algorithm turns $M$ into a regular expression $\alpha$, where the regular expression simplification function $\text{simp} \in \text{Reg} \to \text{Reg}$ is $\text{simplify}(\text{weakSubset})$. Use Forlan to carry out the calls to $\text{simp}$. Also use Forlan to check that your final answer is correct. (Include a transcript of your Forlan session.)
Exercise 3 (60 points)

Define \( \text{Sur} \in \{0,1,2\}^* \times \{0,1,2\}^* \times \{0,1,2\}^* \rightarrow \text{Lan} \) by: for all \( x, y, z \in \{0,1,2\}^* \),
\[ \text{Sur}(x, y, z) = \{ w \in \{0,1,2\}^* \mid \text{for all } u, v \in \{0,1,2\}^*, \text{if } w = uyv, \text{ then } x \text{ is a suffix of } u \text{ and } z \text{ is a prefix of } v \}. \] ("sur" is short for "surround").

(a) Explain how some of the functions/algorithms that we have studied can be used to define a function/algorithm \( \text{surDFA} \in \{0,1,2\}^* \times \{0,1,2\}^* \times \{0,1,2\}^* \rightarrow \text{DFA} \) such that, for all \( x, y, z \in \{0,1,2\}^* \), \( \text{surDFA}(x, y, z) \) is a DFA, with as few states as possible, such that \( L(\text{surDFA}(x, y, z)) = \text{Sur}(x, y, z) \). Prove that your answer is correct. [40 points]

(b) Turn the definition of \( \text{surDFA} \) into an SML/Forlan function

\[
\text{val surDFA} : \text{str} * \text{str} * \text{str} \rightarrow \text{dfa}
\]

You should assume that \( \text{surDFA} \) will only be called with elements of \( \{0,1,2\}^* \). Evaluate the declaration

\[
\text{val dfa} =
\text{surDFA}(_, _, _);
\]

Display \( \text{dfa} \) using \( \text{DFA.output} \). Also draw \( \text{dfa} \), doing your best to make its structure clear. (Include a transcript of your Forlan session.) [20 points]