CIS 570 — Introduction to Formal Language Theory — Fall 2007

Final Examination

Monday, December 10, 11:50 a.m.–1:40 p.m.

Question 1 (25 points)

(a) Let \( X = \{ 0^i1^j2^k3^l \mid i, j, k, l \in \mathbb{N} \text{ and } i + j + k = 2l \} \). Find a grammar \( G \) such that \( L(G) = X \). [20 points]

(b) Draw one or more parse trees \( pt_1, pt_2, \ldots, pt_n \) such that:

- for all \( 1 \leq i \leq n \), \( pt_i \) is valid for \( G \), \( \text{rootLabel} \; pt_i = s_G \) and \( \text{yield} \; pt_i \in \{0, 1, 2, 3\}^* \); and

- each production of \( G \) is used by at least one of the parse trees \( pt_1, pt_2, \ldots, pt_n \).

Say what \( \text{yield} \; pt_1, \text{yield} \; pt_2, \ldots, \text{yield} \; pt_n \) are. [5 points]

Question 2 (20 points)

Let \( X = \{ w \in \{0, 1\}^* \mid \text{if 000011 is a substring of } w, \text{ then 110000 is a substring of } w \} \). Carefully explain how you could use Forlan to find a DFA \( M \), with as few states as possible, such that \( L(M) = X \). You should try to make Forlan do as much of the work of finding \( M \) as possible.

Question 3 (45 points)

Let \( X = \{ w \in \{0, 1\}^* \mid \text{neither } 00 \text{ nor } 11 \text{ is a substring of } w \} \).

(a) Find a DFA \( M \) such that \( L(M) = X \). [10 points]

(b) Prove that your answer to part (a) is correct. [35 points]

Question 4 (10 points)

Compare and contrast the Pumping Lemma for Regular Languages (PLRL) with the Pumping Lemma for Context-free Languages (PLCFL). In particular, explain why one of the pumping lemmas is easier to use than the other.