CIS 570 — Introduction to Formal Language Theory — Fall 2007

Final Examination

Monday, December 10, 11:50 a.m.–1:40 p.m.

Question 1 (25 points)

(a) Let $X = \{\, \mathsf{0}^i \mathsf{1}^j \mathsf{2}^k \mathsf{3}^l \mid i, j, k, l \in \mathbb{N} \text{ and } i + j + k = 2l \,\}$. Find a grammar G such that $L(G) = X$. [20 points]

(b) Draw one or more parse trees pt_1, pt_2, ..., pt_n such that:

- for all $1 \leq i \leq n$, pt_i is valid for G, $\textbf{rootLabel}\, pt_i = s_G$ and $\textbf{yield}\, pt_i \in \{\mathsf{0}, \mathsf{1}, \mathsf{2}, \mathsf{3}\}^*$; and
- each production of G is used by at least one of the parse trees pt_1, pt_2, ..., pt_n.

Say what $\textbf{yield}\, pt_1$, $\textbf{yield}\, pt_2$, ..., $\textbf{yield}\, pt_n$ are. [5 points]

Question 2 (20 points)

Let $X = \{\, w \in \{\mathsf{0}, \mathsf{1}\}^* \mid \text{if } 000011 \text{ is a substring of } w, \text{ then } 110000 \text{ is a substring of } w \,\}$. Carefully explain how you could use Forlan to find a DFA M, with as few states as possible, such that $L(M) = X$. You should try to make Forlan do as much of the work of finding M as possible.

Question 3 (45 points)

Let $X = \{\, w \in \{\mathsf{0}, \mathsf{1}\}^* \mid \text{neither } \mathsf{00} \text{ nor } \mathsf{11} \text{ is a substring of } w \,\}$.

(a) Find a DFA M such that $L(M) = X$. [10 points]

(b) Prove that your answer to part (a) is correct. [35 points]

Question 4 (10 points)

Compare and contrast the Pumping Lemma for Regular Languages (PLRL) with the Pumping Lemma for Context-free Languages (PLCFL). In particular, explain why one of the pumping lemmas is easier to use than the other.