CIS 570 — Introduction to Formal Language Theory — Fall 2007

Mid-term Examination

Wednesday, October 31, 9:30–10:20 a.m.

Question 1 (25 points)

Find a finite automaton M such that $L(M) = (\{0\}^*\{11\}\{222\}^*)^*$.

Question 2 (15 points)

Let

$$X = \{\, w \in \{0,1\}^* \mid \text{for all } x, y \in \{0,1\}^*,\ \text{if } w = x00y,\ \text{then } 11 \text{ is a substring of } y \,\},$$
$$Y = \{\, w \in \{0,1\}^* \mid \text{for all } x, y \in \{0,1\}^*,\ \text{if } w = x00y,\ \text{then } 11 \text{ is a prefix of } y \,\}.$$

For each of the following five statements, say whether the statement is true or false. You *don't* have to prove that your claims are correct.

(a) $X \subsetneq Y$ (X is a proper subset of Y); [3 points]

(b) $Y \subsetneq X$ (Y is a proper subset of X); [3 points]

(c) $X = Y$; [3 points]

(d) $X \cap Y = \emptyset$; [3 points]

(e) $X \cap Y \neq \emptyset$. [3 points]

Question 3 (60 points)

Given $w \in \{0,1\}^*$, we write $\mathbf{zeros}(w)$ for the number of occurrences of 0 in w. For example, $\mathbf{zeros}(01101) = 2$. Let $X = \{\, w \in \{0,1\}^* \mid \mathbf{zeros}(w) \text{ is even and } 1 \text{ is not a prefix of } w \,\}$. For example, $01101 \in X$, but $111 \notin X$, even though $\mathbf{zeros}(111) = 0$ is even.

(a) Find a regular expression α such that $L(\alpha) = X$. [20 points]

(b) Prove that your answer to Part (a) is correct. [40 points]