

Simple Extensions

We take the simply typed lambda calculus—plus booleans and natural numbers—as a base, and consider a number of additions.

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The Unit Type

New syntactic forms

$t ::= \dots$	terms:
unit	constant unit
$v ::= \dots$	values:
unit	constant unit
$T ::= \dots$	types:
Unit	unit type

New typing rules: $\boxed{\Gamma \vdash t : T}$

$\Gamma \vdash \text{unit} : \text{Unit}$ (T-Unit)

New derived forms:

$t_1; t_2 \stackrel{\text{def}}{=} (\lambda x : \text{Unit}. t_2) t_1$, where $x \notin FV(t_2)$.

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Ascription

New syntactic forms

$t ::= \dots$ terms:
 $t \text{ as } T$ ascription

New evaluation rules : $t \rightarrow t'$

$v_1 \text{ as } T \rightarrow v_1$ (E-Ascribe)

$$\frac{t_1 \rightarrow t'_1}{t_1 \text{ as } T \rightarrow t'_1 \text{ as } T}$$
 (E-Ascribe1)

New typing rules: $\Gamma \vdash t : T$

$$\frac{\Gamma \vdash t_1 : T}{\Gamma \vdash t_1 \text{ as } T : T}$$
 (T-Ascribe)

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Let Bindings

New syntactic forms

$t ::= \dots$ terms:
 $\text{let } x = t \text{ in } t$ let binding

New evaluation rules : $t \rightarrow t'$

$\text{let } x = v_1 \text{ in } t_2 \rightarrow [x \mapsto v_1]t_2$ (E-LetV)

$$\frac{t_1 \rightarrow t'_1}{\text{let } x = t_1 \text{ in } t_2 \rightarrow \text{let } x = t'_1 \text{ in } t_2}$$
 (E-Let)

New typing rules: $\Gamma \vdash t : T$

$$\frac{\Gamma \vdash t_1 : T_1 \quad \Gamma, x : T_1 \vdash t_2 : T_2}{\Gamma \vdash \text{let } x = t_1 \text{ in } t_2 : T_2}$$
 (T-Let)

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Pairs

New syntactic forms

$t ::= \dots$	terms:
$\{t, t\}$	pair
$t.1$	first projection
$t.2$	second projection
$v ::= \dots$	values:
$\{v, v\}$	pair value
$T ::= \dots$	types:
$T \times T$	product type

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Pairs (Cont.)

New evaluation rules : $t \rightarrow t'$

$$\frac{t_1 \rightarrow t'_1}{\{t_1, t_2\} \rightarrow \{t'_1, t_2\}} \quad (\text{E-Pair1})$$

$$\frac{t_2 \rightarrow t'_2}{\{v_1, t_2\} \rightarrow \{v_1, t'_2\}} \quad (\text{E-Pair2})$$

$$\frac{t_1 \rightarrow t'_1}{t_1.1 \rightarrow t'_1.1} \quad (\text{E-Proj1})$$

$$\frac{t_1 \rightarrow t'_1}{t_1.2 \rightarrow t'_1.2} \quad (\text{E-Proj2})$$

$$\{v_1, v_2\}.1 \rightarrow v_1 \quad (\text{E-PairBeta1})$$

$$\{v_1, v_2\}.2 \rightarrow v_2 \quad (\text{E-PairBeta2})$$

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Pairs (Cont.)

New typing rules: $\boxed{\Gamma \vdash t : T}$

$$\frac{\Gamma \vdash t_1 : T_1 \quad \Gamma \vdash t_2 : T_2}{\Gamma \vdash \{t_1, t_2\} : T_1 \times T_2} \quad (\text{T-Pair})$$

$$\frac{\Gamma \vdash t_1 : T_{11} \times T_{12}}{\Gamma \vdash t_{1.1} : T_{11}} \quad (\text{T-Proj1})$$

$$\frac{\Gamma \vdash t_1 : T_{11} \times T_{12}}{\Gamma \vdash t_{1.2} : T_{12}} \quad (\text{T-Proj2})$$

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Records

New syntactic forms

$t ::= \dots$ terms:

$\{l_1 = t_1, \dots, l_n = t_n\}$ record

$t.l$ projection

$v ::= \dots$ values:

$\{l_1 = v_1, \dots, l_n = v_n\}$ record value

$T ::= \dots$ types:

$\{l_1 : T_1, \dots, l_n : T_n\}$ type of records

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Records (Cont.)

New evaluation rules : $t \rightarrow t'$

$$\{l_1 = v_1, \dots, l_n = v_n\}.l_j \rightarrow v_j \quad (\text{E-ProjRcd})$$

$$\frac{t_1 \rightarrow t'_1}{t_1.l \rightarrow t'_1.l} \quad (\text{E-Proj})$$

$$\frac{t_j \rightarrow t'_j}{\{l_1 = v_1, \dots, l_{j-1} = v_{j-1}, l_j = t_j, l_{j+1} = t_{j+1}, \dots, l_n = t_n\} \rightarrow \{l_1 = v_1, \dots, l_{j-1} = v_{j-1}, l_j = t'_j, l_{j+1} = t_{j+1}, \dots, l_n = t_n\}} \quad (\text{E-Rcd})$$

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Records (Cont.)

New typing rules: $\Gamma \vdash t : T$

$$\frac{\Gamma \vdash t_1 : T_1 \quad \dots \quad \Gamma \vdash t_n : T_n}{\Gamma \vdash \{l_1 : t_1, \dots, l_n : t_n\} : \{l_1 : T_1, \dots, l_n : T_n\}} \quad (\text{T-Rcd})$$

$$\frac{\Gamma \vdash t_1 : \{l_1 : T_1, \dots, l_n : T_n\}}{\Gamma \vdash t_1.l_j : T_j} \quad (\text{T-Proj})$$

Variants

New syntactic forms

$t ::= \dots$	terms:
$\langle l = t \rangle \text{ as } T$	tagging
$\text{case } t \text{ of } \langle l_1 = x_1 \rangle \Rightarrow t_1 \mid \dots \mid \langle l_n = x_n \rangle \Rightarrow t_n$	case
$v ::= \dots$	values:
$\langle l = v \rangle \text{ as } T$	tagging value
$T ::= \dots$	types:
$\langle l_1 : T_1, \dots, l_n : T_n \rangle$	type of variants

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Variants (Cont.)

New evaluation rules : $t \rightarrow t'$

$\text{case } \langle l_j = v_j \rangle \text{ as } T \text{ of}$
 $\langle l_1 = x_1 \rangle \Rightarrow t_1 \mid \dots \mid \langle l_n = x_n \rangle \Rightarrow t_n \rightarrow$ (E-CaseVariant)
 $[x_j \mapsto v_j]t_j$

$$\frac{t_0 \rightarrow t'_0}{\text{case } t_0 \text{ of}} \quad \text{(E-Case)}$$

$\text{case } t_0 \text{ of}$
 $\langle l_1 = x_1 \rangle \Rightarrow t_1 \mid \dots \mid \langle l_n = x_n \rangle \Rightarrow t_n \rightarrow$
 $\text{case } t'_0 \text{ of}$
 $\langle l_1 = x_1 \rangle \Rightarrow t_1 \mid \dots \mid \langle l_n = x_n \rangle \Rightarrow t_n$

$$\frac{t \rightarrow t'}{\langle l = t \rangle \text{ as } T \rightarrow \langle l = t' \rangle \text{ as } T} \quad \text{(E-Variant)}$$

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Variants (Cont.)

New typing rules: $\boxed{\Gamma \vdash t : T}$

$$\frac{\Gamma \vdash t_j : T_j}{\Gamma \vdash \langle l_j = t_j \rangle \text{ as } \langle l_1 : T_1, \dots, l_n : T_n \rangle : \langle l_1 : T_1, \dots, l_n : T_n \rangle} \quad (\text{T-Variant})$$

$$\frac{\Gamma \vdash t_0 : \langle l_1 : T_1, \dots, l_n : T_n \rangle \quad \Gamma, x_1 : T_1 \vdash t_1 : T \quad \dots \quad \Gamma, x_n : T_n \vdash t_n : T}{\Gamma \vdash \text{case } t_0 \text{ of } \langle l_1 = x_1 \rangle \Rightarrow t_1 \mid \dots \mid \langle l_n = x_n \rangle \Rightarrow t_n : T} \quad (\text{T-Case})$$

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General Recursion

New syntactic forms

$t ::= \dots$ terms:
 $\text{fix } t$ fixed point of t

New evaluation rules : $\boxed{t \rightarrow t'}$

$$\text{fix } (\lambda x : T_1. t_2) \rightarrow [x \mapsto \text{fix } (\lambda x : T_1. t_2)] t_2 \quad (\text{E-FixBeta})$$

$$\frac{t_1 \rightarrow t'_1}{\text{fix } t_1 \rightarrow \text{fix } t'_1} \quad (\text{E-Fix})$$

New typing rules: $\boxed{\Gamma \vdash t : T}$

$$\frac{\Gamma \vdash t_1 : T_1 \rightarrow T_1}{\text{fix } t_1 : T_1} \quad (\text{T-Fix})$$

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