

# References: Types

$T ::=$	types:
$T \rightarrow T$	type of functions
Unit	unit type
Ref $T$	type of reference cells

# References: Terms

## Syntactic forms

$t ::=$	terms:
$x$	variables
$\lambda x: T. t$	abstraction
$t t$	application
$\text{unit}$	constant <b>unit</b>
$\text{ref } t$	reference creation
$!t$	dereference
$t := t$	assignment

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$l$	store location

Store locations  $l$  can only appear in *internal* terms.

# References: Values

## Syntactic forms

$v ::=$	values:
$\lambda x : T. t$	abstraction value
unit	constant unit
$l$	store location

# References: Contexts, Stores and Store Typings

$\Gamma ::=$  contexts:  
 $\emptyset$  empty context  
 $\Gamma, x : T$  term variable binding

$\mu ::=$  stores:  
 $\emptyset$  empty store  
 $\mu, l \mapsto v$  location binding

$\Sigma ::=$  store typings:  
 $\emptyset$  empty store typing  
 $\Sigma, l : T$  location typing

# References: Evaluation

**Evaluation:**  $t \mid \mu \rightarrow t' \mid \mu'$

$$\frac{t_1 \mid \mu \rightarrow t'_1 \mid \mu'}{t_1 t_2 \mid \mu \rightarrow t'_1 t_2 \mid \mu'} \quad (\text{E-App1})$$

$$\frac{t_2 \mid \mu \rightarrow t'_2 \mid \mu'}{v_1 t_2 \mid \mu \rightarrow v_1 t'_2 \mid \mu'} \quad (\text{E-App2})$$

$$(\lambda x: T_{11}. t_{12})v_2 \mid \mu \rightarrow [x \mapsto v_2]t_{12} \mid \mu \quad (\text{E-AppAbs})$$

$$\frac{l \notin \text{dom}(\mu)}{\text{ref } v_1 \mid \mu \rightarrow l \mid (\mu, l \mapsto v_1)} \quad (\text{E-RefV})$$

$$\frac{t_1 \mid \mu \rightarrow t'_1 \mid \mu'}{\text{ref } t_1 \mid \mu \rightarrow \text{ref } t'_1 \mid \mu'} \quad (\text{E-Ref})$$

## References: Evaluation (Cont.)

**Evaluation:**

$$\boxed{t \mid \mu \rightarrow t' \mid \mu'}$$

$$\frac{\mu(l) = v}{!l \mid \mu \rightarrow v \mid \mu} \quad (\text{E-DerefLoc})$$

$$\frac{t_1 \mid \mu \rightarrow t'_1 \mid \mu'}{!t_1 \mid \mu \rightarrow !t'_1 \mid \mu'} \quad (\text{E-Deref})$$

$$l := v_2 \mid \mu \rightarrow \text{unit} \mid [l \mapsto v_2]\mu \quad (\text{E-Assign})$$

$$\frac{t_1 \mid \mu \rightarrow t'_1 \mid \mu'}{t_1 := t_2 \mid \mu \rightarrow t'_1 := t_2 \mid \mu'} \quad (\text{E-Assign1})$$

$$\frac{t_2 \mid \mu \rightarrow t'_2 \mid \mu'}{v_1 := t_2 \mid \mu \rightarrow v_1 := t'_2 \mid \mu'} \quad (\text{E-Assign2})$$



# References: Typing

$$\boxed{\Gamma \mid \Sigma \vdash t : T}$$
$$\frac{x : T \in \Gamma}{\Gamma \mid \Sigma \vdash x : T} \quad (\text{T-Var})$$
$$\frac{\Gamma, x : T_1 \mid \Sigma \vdash t_2 : T_2}{\Gamma \mid \Sigma \vdash \lambda x : T_1. t_2 : T_1 \rightarrow T_2} \quad (\text{T-Abs})$$
$$\frac{\Gamma \mid \Sigma \vdash t_1 : T_{11} \rightarrow T_{12} \quad \Gamma \mid \Sigma \vdash t_2 : T_{11}}{\Gamma \mid \Sigma \vdash t_1 t_2 : T_{12}} \quad (\text{T-App})$$
$$\Gamma \mid \Sigma \vdash \text{unit} : \text{Unit} \quad (\text{T-Unit})$$

# References: Typing

$$\boxed{\Gamma \mid \Sigma \vdash t : T}$$

$$\frac{x : T \in \Gamma}{\Gamma \mid \Sigma \vdash x : T} \quad (\text{T-Var})$$

$$\frac{\Gamma, x : T_1 \mid \Sigma \vdash t_2 : T_2}{\Gamma \mid \Sigma \vdash \lambda x : T_1. t_2 : T_1 \rightarrow T_2} \quad (\text{T-Abs})$$

$$\frac{\Gamma \mid \Sigma \vdash t_1 : T_{11} \rightarrow T_{12} \quad \Gamma \mid \Sigma \vdash t_2 : T_{11}}{\Gamma \mid \Sigma \vdash t_1 t_2 : T_{12}} \quad (\text{T-App})$$

$$\Gamma \mid \Sigma \vdash \text{unit} : \text{Unit} \quad (\text{T-Unit})$$

$\Sigma = \emptyset$  when typing external language.

## References: Typing (Cont.)

$$\Gamma \mid \Sigma \vdash t : T$$

$$\frac{\Sigma(l) = T_1}{\Gamma \mid \Sigma \vdash l : \text{Ref } T_1} \quad (\text{T-Loc})$$

$$\frac{\Gamma \mid \Sigma \vdash t_1 : T_1}{\Gamma \mid \Sigma \vdash \text{ref } t_1 : \text{Ref } T_1} \quad (\text{T-Ref})$$

$$\frac{\Gamma \mid \Sigma \vdash t_1 : \text{Ref } T_{11}}{\Gamma \mid \Sigma \vdash !t_1 : T_{11}} \quad (\text{T-Deref})$$

$$\frac{\Gamma \mid \Sigma \vdash t_1 : \text{Ref } T_{11} \quad \Gamma \mid \Sigma \vdash t_2 : T_{11}}{\Gamma \mid \Sigma \vdash t_1 := t_2 : \text{Unit}} \quad (\text{T-Assign})$$