

# Typed Arithmetic Expressions

## Syntactic Forms

$t ::=$	terms:
true	constant true
false	constant false
if $t$ then $t$ else $t$	conditional
0	constant zero
succ $t$	successor
pred $t$	predecessor
iszero $t$	zero test

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# Typed Arithmetic Expressions: Syntax (Cont.)

## Syntactic Forms

$v ::=$	values:
true	true value
false	false value
$nv$	numeric value
$nv ::=$	numeric values:
0	zero value
succ $nv$	successor value

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## Typed Arithmetic Expressions: Types

$T ::=$  types:  
    Bool      type of booleans  
    Nat      type of natural numbers

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## Typed Arithmetic Expressions: Typing Rules

$t : T$

true : Bool      (T-True)

false : Bool      (T-False)

$\frac{t_1 : \text{Bool} \quad t_2 : T \quad t_3 : T}{\text{if } t_1 \text{ then } t_2 \text{ else } t_3 : T}$       (T-If)

0 : Nat      (T-Zero)

$\frac{t_1 : \text{Nat}}{\text{succ } t_1 : \text{Nat}}$       (T-Succ)

$\frac{t_1 : \text{Nat}}{\text{pred } t_1 : \text{Nat}}$       (T-Pred)

$\frac{t_1 : \text{Nat}}{\text{iszero } t_1 : \text{Bool}}$       (T-Iszero)

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## Typed Arithmetic Expressions: Evaluation

**Evaluation:**  $t \rightarrow t'$

if true then  $t_2$  else  $t_3 \rightarrow t_2$  (E-IfTrue)

if false then  $t_2$  else  $t_3 \rightarrow t_3$  (E-IfFalse)

$$\frac{t_1 \rightarrow t'_1}{\text{if } t_1 \text{ then } t_2 \text{ else } t_3 \rightarrow \text{if } t'_1 \text{ then } t_2 \text{ else } t_3} \quad (\text{E-If})$$

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## Typed Arithmetic Expressions: Evaluation (Cont.)

**Evaluation Rules:**  $t \rightarrow t'$

$$\frac{t_1 \rightarrow t'_1}{\text{succ } t_1 \rightarrow \text{succ } t'_1} \quad (\text{E-Succ})$$

pred 0  $\rightarrow$  0 (E-PredZero)

pred (succ  $nv_1$ )  $\rightarrow$   $nv_1$  (E-PredSucc)

$$\frac{t_1 \rightarrow t'_1}{\text{pred } t_1 \rightarrow \text{pred } t'_1} \quad (\text{E-Pred})$$

iszero 0  $\rightarrow$  true (E-IszeroZero)

iszero (succ  $nv_1$ )  $\rightarrow$  false (E-IszeroSucc)

$$\frac{t_1 \rightarrow t'_1}{\text{iszero } t_1 \rightarrow \text{iszero } t'_1} \quad (\text{E-Iszero})$$

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