

Typed Arithmetic Expressions

Syntactic Forms

$t ::=$	terms:
true	constant true
false	constant false
if t then t else t	conditional
0	constant zero
succ t	successor
pred t	predecessor
iszero t	zero test

Typed Arithmetic Expressions: Syntax (Cont.)

Syntactic Forms

$v ::=$ values:

true true value

false false value

nv numeric value

$nv ::=$ numeric values:

0 zero value

$\text{succ } nv$ successor value

Typed Arithmetic Expressions: Types

$T ::=$ types:
 Bool type of booleans
 Nat type of natural numbers

Typed Arithmetic Expressions: Typing Rules

$t : T$

$\text{true} : \text{Bool}$ (T-True)

$\text{false} : \text{Bool}$ (T-False)

$$\frac{t_1 : \text{Bool} \quad t_2 : T \quad t_3 : T}{\text{if } t_1 \text{ then } t_2 \text{ else } t_3 : T}$$
 (T-If)

$0 : \text{Nat}$ (T-Zero)

$$\frac{t_1 : \text{Nat}}{\text{succ } t_1 : \text{Nat}}$$
 (T-Succ)

$$\frac{t_1 : \text{Nat}}{\text{pred } t_1 : \text{Nat}}$$
 (T-Pred)

$$\frac{t_1 : \text{Nat}}{\text{iszero } t_1 : \text{Bool}}$$
 (T-Iszero)

Typed Arithmetic Expressions: Evaluation

Evaluation: $t \rightarrow t'$

if true then t_2 else $t_3 \rightarrow t_2$ (E-IfTrue)

if false then t_2 else $t_3 \rightarrow t_3$ (E-IfFalse)

$$\frac{t_1 \rightarrow t'_1}{\text{if } t_1 \text{ then } t_2 \text{ else } t_3 \rightarrow \text{if } t'_1 \text{ then } t_2 \text{ else } t_3} \quad (\text{E-If})$$

Typed Arithmetic Expressions: Evaluation (Cont.)

Evaluation Rules: $t \rightarrow t'$

$\frac{t_1 \rightarrow t'_1}{\text{succ } t_1 \rightarrow \text{succ } t'_1}$	(E-Succ)
$\text{pred } 0 \rightarrow 0$	(E-PredZero)
$\text{pred } (\text{succ } nv_1) \rightarrow nv_1$	(E-PredSucc)
$\frac{t_1 \rightarrow t'_1}{\text{pred } t_1 \rightarrow \text{pred } t'_1}$	(E-Pred)
$\text{iszero } 0 \rightarrow \text{true}$	(E-IszeroZero)
$\text{iszero } (\text{succ } nv_1) \rightarrow \text{false}$	(E-IszeroSucc)
$\frac{t_1 \rightarrow t'_1}{\text{iszero } t_1 \rightarrow \text{iszero } t'_1}$	(E-Iszero)