

Simply Typed Lambda Calculus: Types

| | |
|-------------------|-------------------|
| $T ::=$ | types: |
| Bool | type of booleans |
| $T \rightarrow T$ | type of functions |

1

Simply Typed Lambda Calculus: Syntax

Syntactic Forms

| | |
|--|----------------|
| $t ::=$ | terms: |
| true | constant true |
| false | constant false |
| $\text{if } t \text{ then } t \text{ else } t$ | conditional |
| x | variables |
| $\lambda x : T. t$ | abstraction |
| tt | application |

2

Simply Typed Lambda Calculus: Syntax (Cont.)

Syntactic Forms

| | |
|-------------------|-------------------|
| $v ::=$ | values: |
| true | true value |
| false | false value |
| $\lambda x: T. t$ | abstraction value |

3

Simply Typed Lambda Calculus: Evaluation

Evaluation: $t \rightarrow t'$

if true then t_2 else $t_3 \rightarrow t_2$ (E-IfTrue)

if false then t_2 else $t_3 \rightarrow t_3$ (E-IfFalse)

$$\frac{t_1 \rightarrow t'_1}{\text{if } t_1 \text{ then } t_2 \text{ else } t_3 \rightarrow \text{if } t'_1 \text{ then } t_2 \text{ else } t_3}$$
 (E-If)

$$\frac{t_1 \rightarrow t'_1}{t_1 t_2 \rightarrow t'_1 t_2}$$
 (E-App1)

$$\frac{t_2 \rightarrow t'_2}{v_1 t_2 \rightarrow v_1 t'_2}$$
 (E-App2)

$(\lambda x: T_{11}. t_{12})v_2 \rightarrow [x \mapsto v_2]t_{12}$ (E-AppAbs)

4

Simply Typed Lambda Calculus: Typing Contexts

| | |
|-----------------|-----------------------|
| $\Gamma ::=$ | contexts: |
| \emptyset | empty context |
| $\Gamma, x : T$ | term variable binding |

A variable may only appear once in a context. We write $\text{dom}(\Gamma)$ for $\{x \mid x : T \in \Gamma, \text{ for some } T\}$, and $\Gamma(x)$ is the unique T such that $x : T \in \Gamma$, if it exists; otherwise $\Gamma(x)$ is undefined.

We abbreviate $\emptyset, x_1 : T_1, \dots, x_n : T_n$ to $x_1 : T_1, \dots, x_n : T_n$.

5

Simply Typed Lambda Calculus: Typing Rules

$\Gamma \vdash t : T$

| | |
|--|-----------|
| $\Gamma \vdash \text{true} : \text{Bool}$ | (T-True) |
| $\Gamma \vdash \text{false} : \text{Bool}$ | (T-False) |
| $\frac{\Gamma \vdash t_1 : \text{Bool} \quad \Gamma \vdash t_2 : T \quad \Gamma \vdash t_3 : T}{\Gamma \vdash \text{if } t_1 \text{ then } t_2 \text{ else } t_3 : T}$ | (T-If) |
| $\frac{x : T \in \Gamma}{\Gamma \vdash x : T}$ | (T-Var) |
| $\frac{\Gamma, x : T_1 \vdash t_2 : T_2 \quad x \notin \text{dom}(\Gamma)}{\Gamma \vdash \lambda x : T_1. t_2 : T_1 \rightarrow T_2}$ | (T-Abs) |
| $\frac{\Gamma \vdash t_1 : T_{11} \rightarrow T_{12} \quad \Gamma \vdash t_2 : T_{11}}{\Gamma \vdash t_1 t_2 : T_{12}}$ | (T-App) |

We abbreviate $\emptyset \vdash t : T$ to $\vdash t : T$.

6