

Breadth Exam in Formal Language Theory

Spring 2008

Duration: 90 Minutes

Format: Closed-book, Closed-notes

Question 1 (25 points)

Find a context-free grammar G such that $L(G) = \{0^i 1^j 2^k 3^l \mid i, j, k, l \in \mathbb{N} \text{ and either } i = l \text{ or } j = k\}$.

Question 2 (75 points)

Define a function $\mathbf{zz} \in \{0, 1\}^* \rightarrow \mathbb{N}$ by:

- $\mathbf{zz}(\epsilon) = 0$;
- for all $w \in \{0, 1\}^*$,

$$\mathbf{zz}(w0) = \begin{cases} \mathbf{zz}(w) + 1, & \text{if } 0 \text{ is a suffix of } w, \\ \mathbf{zz}(w), & \text{if } 0 \text{ is not a suffix of } w; \end{cases}$$

- for all $w \in \{0, 1\}^*$, $\mathbf{zz}(w1) = \mathbf{zz}(w)$.

(Thus $\mathbf{zz}(w)$ is the number of possibly overlapping occurrences of 00 in w .) Let $X = \{w \in \{0, 1\}^* \mid \mathbf{zz}(w) \text{ is even}\}$. (Thus, e.g., $000 \in X$, since $\mathbf{zz}(000) = 2$ is even.)

(a) Find a DFA M such that $L(M) = X$. [25 points]

(b) Prove that your answer to Part (a) is correct. [50 points]