Breadth Exam in Formal Language Theory
Spring 2010

Duration: 90 Minutes
Format: Closed-book, Closed-notes

Question 1 (25 points)

Find a grammar G such that $L(G) = \{\, \mathsf{0}^i\mathsf{1}^j\mathsf{2}^k\mathsf{3}^l \mid i, j, k, l \in \mathbb{N} \text{ and } i + j = k + l \,\}$.

Question 2 (75 points)

Given a string $w \in \{\mathsf{0}, \mathsf{1}\}^*$, we write $\mathbf{diff}(w)$ ("difference") for

$$\text{the number of } \mathsf{1}\text{'s in } w - 2(\text{the number of } \mathsf{0}\text{'s in } w).$$

Then:

- $\mathbf{diff}(\epsilon) = 0 - 2 \times 0 = 0 - 0 = 0$;
- $\mathbf{diff}(\mathsf{1}) = 1 - 2 \times 0 = 1 - 0 = 1$;
- $\mathbf{diff}(\mathsf{0}) = 0 - 2 \times 1 = 0 - 2 = -2$;
- for all $x, y \in \{\mathsf{0}, \mathsf{1}\}^*$, $\mathbf{diff}(xy) = \mathbf{diff}(x) + \mathbf{diff}(y)$.

For example, $\mathbf{diff}(\mathsf{10010}) = 2 - 2 \times 3 = 2 - 6 = -4$.

Let

$$X = \{\, w \in \{\mathsf{0}, \mathsf{1}\}^* \mid \text{for all prefixes } v \text{ of } w,\ 0 \leq \mathbf{diff}(v) \leq 3 \,\}.$$

(a) Find a DFA M such that $L(M) = X$. [25 points]

(b) Prove that your answer to Part (a) is correct. [50 points]