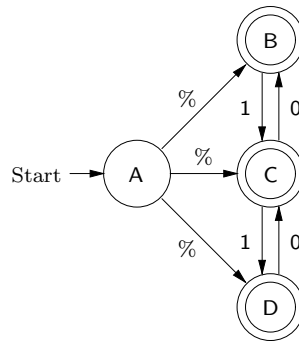


### Exercise Set 3

Due by 4:00 p.m. on Tuesday, October 28

#### Exercise 1 (20 points)

Let  $M$  be the FA



(a) Use Forlan to find and display minimum-length labeled paths explaining why the strings 11001011001, 10011010100 and 00101010110 are accepted by  $M$ . (Include a transcript of your Forlan session.) [10 points]

(b) Describe the language  $L(M)$  as simply as possible, without mentioning  $M$ . [10 points]

**Exercise 2 (80 points)**

Define a function  $\mathbf{diff} \in \{0, 1\}^* \rightarrow \mathbb{Z}$  by: for all  $w \in \{0, 1\}^*$ ,

$$\mathbf{diff} w = \text{the number of 1's in } w - 2(\text{the number of 0's in } w).$$

Thus

- $\mathbf{diff} \epsilon = 0$ ;
- $\mathbf{diff} 0 = -2$ ;
- $\mathbf{diff} 1 = 1$ ;
- for all  $x, y \in \{0, 1\}^*$ ,  $\mathbf{diff}(xy) = \mathbf{diff} x + \mathbf{diff} y$ .

Let  $X = \{w \in \{0, 1\}^* \mid \mathbf{diff} w = 0 \text{ and, for all prefixes } v \text{ of } w, 0 \leq \mathbf{diff} v \leq 3\}$ .

(a) Find an FA  $M$  with as few states as possible such that  $L(M) = X$ . (You can obtain partial credit by finding an  $M$  that has too many states.) [15 points]

(b) Use Forlan to provide some evidence that  $L(M) = X$ , making use of some test cases that are in  $X$ , as well as some that are not in  $X$ . (Include a transcript of your Forlan session. Even if your  $M$  has too many states, you can receive full credit for this part.) [10 points]

(c) Prove that  $L(M) = X$ . (Even if your  $M$  has too many states, you can receive full credit for this part.) [25 points]

(d) Prove that, for all finite automata  $N$ , if  $L(N) = X$ , then  $N$  has at least as many states as  $M$ . [20 points]

(e) Find an FA  $N$  such that  $L(N) = X$ ,  $N$  has the same number of states as  $M$ , but  $N$  and  $M$  are not isomorphic. Use Forlan to check that  $N$  and  $M$  are not isomorphic. (Include a transcript of your Forlan session. Even if your  $M$  has too many states, you can receive full credit for this part.) [10 points]