CIS 570 — Introduction to Formal Language Theory — Fall 2008

Exercise Set 4

Due by 4:00 p.m. on Tuesday, November 11

Exercise 1 (35 points)

(a) Define functions $\mathbf{zo}, \mathbf{oz} \in \{0, 1\}^* \to \mathbb{N}$ by recursion:

- $\mathbf{zo}\,\% = 0$,
- for all $w \in \{0, 1\}^*$,

$$\mathbf{zo}(w1) = \begin{cases} 0, & \text{if } w = \%, \\ \mathbf{zo}\,w, & \text{if 1 is a suffix of } w, \\ \mathbf{zo}\,w + 1, & \text{if 0 is a suffix of } w, \end{cases}$$

- for all $w \in \{0, 1\}^*$, $\mathbf{zo}(w0) = \mathbf{zo}\,w$

and

- $\mathbf{oz}\,\% = 0$,
- for all $w \in \{0, 1\}^*$,

$$\mathbf{oz}(w0) = \begin{cases} 0, & \text{if } w = \%, \\ \mathbf{oz}\,w, & \text{if 0 is a suffix of } w, \\ \mathbf{oz}\,w + 1, & \text{if 1 is a suffix of } w, \end{cases}$$

- for all $w \in \{0, 1\}^*$, $\mathbf{oz}(w1) = \mathbf{oz}\,w$.

Thus $\mathbf{zo}\,w$ is the number of occurrences of 01 in w, and $\mathbf{oz}\,w$ is the number of occurrences of 10 in w. E.g., $\mathbf{zo}(1001) = \mathbf{oz}(1001) = 1$.

Define

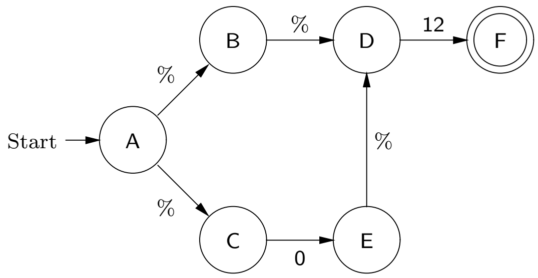
$$X = \{\, w \in \{0, 1\}^* \mid \mathbf{zo}\,w \text{ is even or } \mathbf{oz}\,w \text{ is even}\,\}.$$

Find a DFA M such that $L(M) = X$. [10 points]

(b) Prove that your answer to part (a) is correct. [25 points]

Exercise 2 (50 points)

Let M_1 be the finite automaton



(a) Find a regular expression that is turned by our regular expression-to-FA conversion algorithm into an FA that is isomorphic to M_1. Use Forlan to check that your answer is correct. (Here, and in the rest of this exercise, always include a Forlan transcript.) [12 points]

(b) Give a step-by-step explanation of how our FA-to-EFA conversion algorithm turns M_1 into an EFA, M_2. Draw M_2 and use Forlan to check that your final answer is correct. [12 points]

(c) Use the Forlan function `EFA.renameStatesCanonically` to turn M_2 into an EFA, M_3, that is isomorphic to M_2, but whose states are A, B, C, etc. Draw M_3. [1 point]

(d) Give a step-by-step explanation of how our EFA-to-NFA conversion algorithm turns M_3 into an NFA, M_4. Draw M_4 and use Forlan to check that your final answer is correct. [12 points]

(e) Give a step-by-step explanation of how our NFA-to-DFA conversion algorithm turns M_4 into a DFA, M_5. Draw M_5 and use Forlan to check that your final answer is correct. [12 points]

(f) Use the Forlan function `DFA.renameStatesCanonically` to turn M_5 into a DFA, M_6, that is isomorphic to M_5 but whose states are A, B, C, etc. Draw M_6. [1 point]

Exercise 3 (15 points)

Either prove or disprove the following statement:

> For all $\mathit{simp} \in \mathbf{Reg} \to \mathbf{Reg}$ and $M \in \mathbf{FA}$, if, for all $\alpha \in \mathbf{Reg}$, $L(\mathit{simp}\,\alpha) = L(\alpha)$ and $\mathbf{alphabet}(\mathit{simp}\,\alpha) \subseteq \mathbf{alphabet}\,\alpha$, then $\mathbf{alphabet}(\mathbf{faToReg}\,\mathit{simp}\,M) = \mathbf{alphabet}\,M$.