CIS 570 — Introduction to Formal Language Theory — Fall 2008

Exercise Set 6

Due by 4:00 p.m. on Tuesday, December 9

Exercise 1 (70 points)

Let

$$X = \{\, \mathsf{0}^i \mathsf{1}^j \mathsf{2}^k \mid i, j, k \in \mathbb{N} \text{ and either } i \neq j \text{ or } j \neq k \,\}.$$

(a) Find a grammar G such that $L(G) = X$. [20 points]

(b) Find parse trees that are valid for G, whose root labels are s_G and whose yields are the strings 0012, 00112222 and 011222. Use Forlan to check that your answers are correct. (Include a transcript of your Forlan session.) [5 points]

(c) Use Forlan to provide some additional evidence that $L(G) = X$, making use of some test cases that are in X, as well as some that are not in X. (Include a transcript of your Forlan session.) [10 points]

(d) Prove that your answer to Part (a) is correct. [20 points]

(e) Either prove or disprove the following statement:

> X is regular.

[15 points]

Exercise 2 (30 points)

Given $w \in \{\mathtt{0}, \mathtt{1}\}^*$, we write $\mathbf{zeros}\, w$ for the number of occurrences of 0 in w, and $\mathbf{ones}\, w$ for the number of occurrence of 1 in w. Let

$$X = \{\, w \in \{\mathtt{0}, \mathtt{1}\}^* \mid \mathbf{zeros}\, w \;\mathbf{mod}\; 3 = 1 \text{ and } \mathbf{ones}\, w \;\mathbf{mod}\; 3 = 2 \,\},$$
$$Y = \{\, w \in X \mid \text{ there is no proper prefix } v \text{ of } w \text{ such that } v \in X \,\}.$$

E.g.:

- 110 $\in X$, because $1 \;\mathbf{mod}\; 3 = 1$ and $2 \;\mathbf{mod}\; 3 = 2$.
- 110100110 $\in X$, because $4 \;\mathbf{mod}\; 3 = 1$ and $5 \;\mathbf{mod}\; 3 = 2$.
- Thus 110 $\in Y$, because it's in X and none of its proper prefixes (%, 1 and 11) are in X.
- But 110100110 $\not\in Y$, even though it's in X, because it has 110 $\in X$ as a proper prefix.

(a) Use Forlan to find a DFA M, with as few states as possible, such that $L(M) = Y$. Output M using `DFA.output`, and draw M, making its structure as clear as possible. (Include a transcript of your Forlan session.) [20 points]

(b) For all $n \in \mathbb{N}$, define

$$Y_n = \{\, w \in Y \mid |w| = n \,\}.$$

Use Forlan to calculate $|Y_n|$ for all $n \in [0 : 25]$. (Include a transcript of your Forlan session.) [10 points]