

Exercise Set 6

Due by 4:00 p.m. on Tuesday, December 4

Exercise 1 (70 points)

Let

$$X = \{0^i 1^j 2^k 3^l \mid i, j, k, l \in \mathbb{N} \text{ and } i + j = k + l\}.$$

- (a) Find a grammar G such that $L(G) = X$. [20 points]
- (b) Find parse trees that are valid for G , whose root labels are s_G and whose yields are the strings 001223 and 00112333. Use Forlan to check that your answers are correct. (Include a transcript of your Forlan session.) [5 points]
- (c) Use Forlan to provide some additional evidence that $L(G) = X$, making use of some test cases that are in X , as well as some that are not in X . (Include a transcript of your Forlan session.) [10 points]
- (d) Prove that your answer to Part (a) is correct. [20 points]
- (e) Prove that X is not regular. [15 points]

Exercise 2 (30 points)

Define a function $\mathbf{diff} \in \{0, 1\}^* \rightarrow \mathbb{Z}$ by: for all $w \in \{0, 1\}^*$,

$$\mathbf{diff}(w) = \text{the number of 1's in } w - \text{the number of 0's in } w.$$

Thus:

- $\mathbf{diff}(\%) = 0$;
- $\mathbf{diff}(0) = -1$;
- $\mathbf{diff}(1) = 1$;
- for all $x, y \in \{0, 1\}^*$, $\mathbf{diff}(xy) = \mathbf{diff}(x) + \mathbf{diff}(y)$.

Define $\mathbf{AllSubGood} \in \mathbb{N} \rightarrow \mathbf{Lan}$ by: for all $n \in \mathbb{N}$, $\mathbf{AllSubGood}(n) = \{w \in \{0, 1\}^* \mid \text{for all substrings } v \text{ of } w, |\mathbf{diff}(v)| \leq n\}$.

(a) Define a Forlan/SML function

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val allSubGoodDFA : int -> dfa
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such that, for all $n \in \mathbb{N}$, $\mathbf{allSubGoodDFA } n$ is a DFA with as few states as possible that accepts $\mathbf{AllSubGood}(n)$. (You don't need to worry about what $\mathbf{allSubGoodDFA}$ does when called with a negative number.) Carefully document your code. [20 points]

(b) Use your $\mathbf{allSubGoodDFA}$ function to generate and display (using $\mathbf{DFA.output}$) DFAs accepting the languages $\mathbf{AllSubGood}(2)$ and $\mathbf{AllSubGood}(3)$. Draw each of these machines as clearly as possible. (Include a transcript of your Forlan session.) [10 points]