

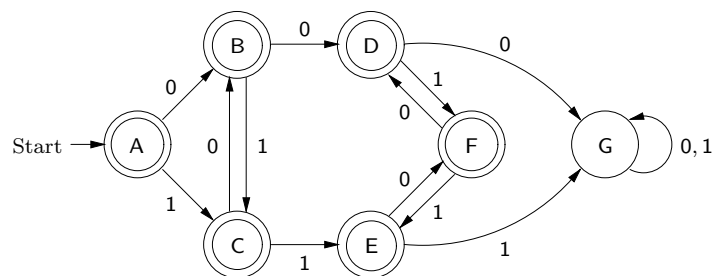
CS 591 S2—Formal Language Theory: Integrating Experimentation and Proof—Fall 2019

Problem Set 4

Model Answers

Problem 1

M is



Problem 2

First, we put the description of M in the file `ps4-dfa`:

```
{states} A, B, C, D, E, F, G
{start state} A
{accepting states} A, B, C, D, E, F
{transitions}
A, 0 -> B; A, 1 -> C;
B, 0 -> D; B, 1 -> C;
C, 0 -> B; C, 1 -> E;
D, 0 -> G; D, 1 -> F;
E, 0 -> F; E, 1 -> G;
F, 0 -> D; F, 1 -> E;
G, 0 -> G; G, 1 -> G
```

We then input M into Forlan, binding it to `dfa`:

```
- val dfa = DFA.input "ps4-dfa";
val dfa = - : dfa
```

Next, we put a testing harness in the file `ps4-p2.sml`:

```
(* val zero : sym
   val one  : sym

   the symbols 0 and 1 *)
```

```

val zero = Sym.fromString "0";
val one  = Sym.fromString "1";

(* val diff : str -> int

   the diff function, assuming input is a string over alphabet {0, 1}
   *)

fun diff (nil : str) = 0
  | diff (b :: bs)  =
    if Sym.equal(b, zero)
    then ~1 + diff bs
    else 1 + diff bs;

(* val allSubstringsGood : str -> bool

   tests whether all substrings of a string over alphabet {0, 1} have
   diff's between ~2 and 2, inclusive *)

fun allSubstringsGood x =
  Set.all
  (fn y =>
    let val n = diff y
        in ~2 <= n andalso n <= 2 end)
  (StrSet.substrings x);

(* val upto : int -> str set

   if n >= 0, then upto n returns all strings over alphabet {0, 1} of
   length no more than n *)

fun upto 0 : str set = Set.sing nil
  | upto n          =
    let val xs = upto(n - 1)
        val ys = Set.filter (fn x => length x = n - 1) xs
    in StrSet.union
      (xs, StrSet.concat(StrSet.fromString "0, 1", ys))
    end;

(* val partition : int -> str set * str set

   if n >= 0, then partition n returns (xs, ys) where:

   xs is all elements of upto n that are in X; and

   ys is all elements of upto n that are not in X *)

fun partition n = Set.partition allSubstringsGood (upto n);

```

```
(* val test = fn : int -> dfa -> str option * str option

if n >= 0, then test n returns a function f such that, for all DFAs
dfa', f dfa' returns a pair (xOpt, yOpt) such that:

If there is an element of {0, 1}* of length no more than n that
is in X but is not by accepted dfa', then xOpt = SOME x for some
such x; otherwise, xOpt = NONE.

If there is an element of {0, 1}* of length no more than n that
is not in X but is by accepted dfa', then yOpt = SOME y for some
such y; otherwise, yOpt = NONE. *)

fun test n =
  let val (goods, bads) = partition n
  in fn dfa =>
    let val determAccepted = DFA.determAccepted dfa
        val failAccOpt     = Set.position (not o determAccepted) goods
        val failNotAccOpt  = Set.position determAccepted bads
    in ((case failAccOpt of
         NONE    => NONE
         | SOME i => SOME(List.sub(Set.toList goods, i))),
       (case failNotAccOpt of
         NONE    => NONE
         | SOME i => SOME(List.sub(Set.toList bads, i))))
    end
  end;
end;
```

We then load the testing harness into Forlan:

```
- use "ps4-p2.sml";
[opening ps4-p2.sml]
val zero = - : sym
val one  = - : sym
val diff = fn : str -> int
val allSubstringsGood = fn : str -> bool
val upto = fn : int -> str set
val partition = fn : int -> str set * str set
val test = fn : int -> dfa -> str option * str option
val it = () : unit
```

Finally, we test M on all $65535 (2^{16} - 1)$ strings of length up to 15 as follows:

```
- val doit = test 15;
val doit = fn : dfa -> str option * str option
- doit dfa;
val it = (NONE,NONE) : str option * str option
```

Problem 3

Define a function **dsfxs** (for “difs of suffixes”) from $\{0,1\}^*$ to $\mathcal{P}\mathbb{Z}$ by: for all $w \in \{0,1\}^*$,

$$\mathbf{dsfxs} w = \{ \mathbf{diff} v \mid v \text{ is a suffix of } w \}.$$

Given $X \subseteq \mathbb{Z}$, we write

$$X^+ = \{ n + 1 \mid n \in X \}, \text{ and}$$

$$X^- = \{ n - 1 \mid n \in X \}.$$

Thus, we have that, for all $X \subseteq \mathbb{Z}$:

- $n \in X$ iff $n + 1 \in X^+$, for all $n \in \mathbb{Z}$;
- $n - 1 \in X$ iff $n \in X^+$, for all $n \in \mathbb{Z}$;
- $n \in X$ iff $n - 1 \in X^-$, for all $n \in \mathbb{Z}$; and
- $n + 1 \in X$ iff $n \in X^-$, for all $n \in \mathbb{Z}$.

Lemma PS4.3.1

- (1) $\% \in X$.
- (2) For all $w \in \{0,1\}^*$, if $w \in X$ and $-2 \notin \mathbf{dsfxs} w$, then $w0 \in X$.
- (3) For all $w \in \{0,1\}^*$, if $w \in X$ and $2 \notin \mathbf{dsfxs} w$, then $w1 \in X$.

Proof.

- (1) Follows since $\%$ is the only substring of $\%$, $\mathbf{diff} \% = 0$ and $-2 \leq 0 \leq 2$.
- (2) Suppose $w \in \{0,1\}^*$, $w \in X$ and $-2 \notin \mathbf{dsfxs} w$. To see that $w0 \in X$, suppose v is a substring of $w0$. We must show that $-2 \leq \mathbf{diff} v \leq 2$. There are two cases to consider (if $v = \%$, then v is a substring of w).
 - Suppose v is a substring of w . Then $-2 \leq \mathbf{diff} v \leq 2$, since $w \in X$.
 - Suppose $v = u0$, for a suffix u of w . Thus $\mathbf{diff} v = \mathbf{diff}(u0) = \mathbf{diff} u + \mathbf{diff} 0 = \mathbf{diff} u - 1$. Because $w \in X$ and u is a substring of w , we have that $-2 \leq \mathbf{diff} u \leq 2$. But $-2 \notin \mathbf{dsfxs} w$, and so $\mathbf{diff} u \neq -2$. Hence $-1 \leq \mathbf{diff} u \leq 2$. Thus $-1 + -1 \leq \mathbf{diff} u - 1 \leq 2 + -1$, so that $-2 = -1 + -1 \leq \mathbf{diff} v \leq 2 + -1 = 1 \leq 2$.
- (3) Suppose $w \in \{0,1\}^*$, $w \in X$ and $2 \notin \mathbf{dsfxs} w$. To see that $w1 \in X$, suppose v is a substring of $w1$. We must show that $-2 \leq \mathbf{diff} v \leq 2$. There are two cases to consider (if $v = \%$, then v is a substring of w).
 - Suppose v is a substring of w . Then $-2 \leq \mathbf{diff} v \leq 2$, since $w \in X$.
 - Suppose $v = u1$, for a suffix u of w . Thus $\mathbf{diff} v = \mathbf{diff}(u1) = \mathbf{diff} u + \mathbf{diff} 1 = \mathbf{diff} u + 1$. Because $w \in X$ and u is a substring of w , we have that $-2 \leq \mathbf{diff} u \leq 2$. But $2 \notin \mathbf{dsfxs} w$, and so $\mathbf{diff} u \neq 2$. Hence $-2 \leq \mathbf{diff} u \leq 1$. Thus $-2 + 1 \leq \mathbf{diff} u + 1 \leq 1 + 1$, so that $-2 \leq -1 = -2 + 1 \leq \mathbf{diff} v \leq 1 + 1 = 2$.

□

Lemma PS4.3.2

- (1) $\mathbf{dsfxs} \% = \{0\}$.
- (2) For all $w \in \{0, 1\}^*$, $\mathbf{dsfxs}(w0) = (\mathbf{dsfxs} w)^- \cup \{0\}$.
- (3) For all $w \in \{0, 1\}^*$, $\mathbf{dsfxs}(w1) = (\mathbf{dsfxs} w)^+ \cup \{0\}$.

Proof.

- (1) We show that $\mathbf{dsfxs} \% \subseteq \{0\} \subseteq \mathbf{dsfxs} \%$.

($\mathbf{dsfxs} \% \subseteq \{0\}$) Suppose $n \in \mathbf{dsfxs} \%$. Then $n = \mathbf{diff} v$ for some suffix v of $\%$. Hence $v = \%$, so that $n = \mathbf{diff} v = \mathbf{diff} \% = 0 \in \{0\}$.

($\{0\} \subseteq \mathbf{dsfxs} \%$) Since $\%$ is a suffix of $\%$, we have that $0 = \mathbf{diff} \% \in \mathbf{dsfxs} \%$. Hence $\{0\} \subseteq \mathbf{dsfxs} \%$.

- (2) Suppose $w \in \{0, 1\}^*$. We show that $\mathbf{dsfxs}(w0) \subseteq (\mathbf{dsfxs} w)^- \cup \{0\} \subseteq \mathbf{dsfxs}(w0)$.

($\mathbf{dsfxs}(w0) \subseteq (\mathbf{dsfxs} w)^- \cup \{0\}$) Suppose $n \in \mathbf{dsfxs}(w0)$. Hence $n = \mathbf{diff} v$ for some suffix v of $w0$. There are two cases to consider.

- Suppose $v = \%$. Then $n = \mathbf{diff} v = \mathbf{diff} \% = 0 \in \{0\} \subseteq (\mathbf{dsfxs} w)^- \cup \{0\}$.
- Suppose $v = u0$, for some suffix u of w . Thus $n = \mathbf{diff} v = \mathbf{diff}(u0) = \mathbf{diff} u + \mathbf{diff} 0 = \mathbf{diff} u + -1 = \mathbf{diff} u - 1$. Since u is a suffix of w , we have that $\mathbf{diff} u \in \mathbf{dsfxs} w$, so that $\mathbf{diff} u - 1 \in (\mathbf{dsfxs} w)^-$. Hence $n = \mathbf{diff} u - 1 \in (\mathbf{dsfxs} w)^- \subseteq (\mathbf{dsfxs} w)^- \cup \{0\}$.

(($(\mathbf{dsfxs} w)^- \cup \{0\} \subseteq \mathbf{dsfxs}(w0)$) Suppose $n \in (\mathbf{dsfxs} w)^- \cup \{0\}$. There are two cases to consider.

- Suppose $n \in (\mathbf{dsfxs} w)^-$. Thus $n + 1 \in \mathbf{dsfxs} w$, so that $n + 1 = \mathbf{diff} v$, for some suffix v of w . Thus $v0$ is a suffix of $w0$, so that $n = \mathbf{diff} v - 1 = \mathbf{diff} v + -1 = \mathbf{diff} v + \mathbf{diff} 0 = \mathbf{diff}(v0) \in \mathbf{dsfxs}(w0)$.
- Suppose $n \in \{0\}$. Since $\%$ is a suffix of $w0$, we have that $n = 0 = \mathbf{diff} \% \in \mathbf{dsfxs}(w0)$.

- (3) Suppose $w \in \{0, 1\}^*$. We show that $\mathbf{dsfxs}(w1) \subseteq (\mathbf{dsfxs} w)^+ \cup \{0\} \subseteq \mathbf{dsfxs}(w1)$.

($\mathbf{dsfxs}(w1) \subseteq (\mathbf{dsfxs} w)^+ \cup \{0\}$) Suppose $n \in \mathbf{dsfxs}(w1)$. Hence $n = \mathbf{diff} v$ for some suffix v of $w1$. There are two cases to consider.

- Suppose $v = \%$. Then $n = \mathbf{diff} v = \mathbf{diff} \% = 0 \in \{0\} \subseteq (\mathbf{dsfxs} w)^+ \cup \{0\}$.
- Suppose $v = u1$, for some suffix u of w . Thus $n = \mathbf{diff} v = \mathbf{diff}(u1) = \mathbf{diff} u + \mathbf{diff} 1 = \mathbf{diff} u + 1$. Since u is a suffix of w , we have that $\mathbf{diff} u \in \mathbf{dsfxs} w$, so that $\mathbf{diff} u + 1 \in (\mathbf{dsfxs} w)^+$. Hence $n = \mathbf{diff} u + 1 \in (\mathbf{dsfxs} w)^+ \subseteq (\mathbf{dsfxs} w)^+ \cup \{0\}$.

(($(\mathbf{dsfxs} w)^+ \cup \{0\} \subseteq \mathbf{dsfxs}(w1)$) Suppose $n \in (\mathbf{dsfxs} w)^+ \cup \{0\}$. There are two cases to consider.

- Suppose $n \in (\mathbf{dsfxs} w)^+$. Thus $n - 1 \in \mathbf{dsfxs} w$, so that $n - 1 = \mathbf{diff} v$, for some suffix v of w . Thus $v1$ is a suffix of $w1$, so that $n = \mathbf{diff} v + 1 = \mathbf{diff} v + \mathbf{diff} 1 = \mathbf{diff}(v1) \in \mathbf{dsfxs}(w1)$.

- Suppose $n \in \{0\}$. Since $\%$ is a suffix of $w1$, we have that $n = 0 = \mathbf{diff} \% \in \mathbf{dsfxs}(w1)$.

□

Lemma PS4.3.3

- (A) For all $w \in \Lambda_A$, $w \in X$ and $\mathbf{dsfxs} w = \{0\}$.
- (B) For all $w \in \Lambda_B$, $w \in X$ and $\mathbf{dsfxs} w = \{-1, 0\}$.
- (C) For all $w \in \Lambda_C$, $w \in X$ and $\mathbf{dsfxs} w = \{0, 1\}$.
- (D) For all $w \in \Lambda_D$, $w \in X$ and $\mathbf{dsfxs} w = \{-2, -1, 0\}$.
- (E) For all $w \in \Lambda_E$, $w \in X$ and $\mathbf{dsfxs} w = \{0, 1, 2\}$.
- (F) For all $w \in \Lambda_F$, $w \in X$ and $\mathbf{dsfxs} w = \{-1, 0, 1\}$.
- (G) For all $w \in \Lambda_G$, $w \notin X$.

Proof. We proceed by induction on Λ . There are 15 (1 plus the number of transitions) parts to show.

(empty string) We must show that $\% \in X$ and $\mathbf{dsfxs} \% = \{0\}$, which follows by Lemma PS4.3.1(1) and Lemma PS4.3.2(1).

(A, 0 \rightarrow B) Suppose $w \in \Lambda_A$, and assume the inductive hypothesis: $w \in X$ and $\mathbf{dsfxs} w = \{0\}$. We must show that $w0 \in X$ and $\mathbf{dsfxs}(w0) = \{-1, 0\}$. Since $w \in X$ and $-2 \notin \mathbf{dsfxs} w$, Lemma PS4.3.1(2) tells us that $w0 \in X$. And, by Lemma PS4.3.2(2), we have that $\mathbf{dsfxs}(w0) = (\mathbf{dsfxs} w)^- \cup \{0\} = \{0\}^- \cup \{0\} = \{-1\} \cup \{0\} = \{-1, 0\}$.

(A, 1 \rightarrow C) Suppose $w \in \Lambda_A$, and assume the inductive hypothesis: $w \in X$ and $\mathbf{dsfxs} w = \{0\}$. We must show that $w1 \in X$ and $\mathbf{dsfxs}(w1) = \{0, 1\}$. Since $w \in X$ and $2 \notin \mathbf{dsfxs} w$, Lemma PS4.3.1(3) tells us that $w1 \in X$. And, by Lemma PS4.3.2(3), we have that $\mathbf{dsfxs}(w1) = (\mathbf{dsfxs} w)^+ \cup \{0\} = \{0\}^+ \cup \{0\} = \{1\} \cup \{0\} = \{0, 1\}$.

(B, 0 \rightarrow D) Suppose $w \in \Lambda_B$, and assume the inductive hypothesis: $w \in X$ and $\mathbf{dsfxs} w = \{-1, 0\}$. We must show that $w0 \in X$ and $\mathbf{dsfxs}(w0) = \{-2, -1, 0\}$. Since $w \in X$ and $-2 \notin \mathbf{dsfxs} w$, Lemma PS4.3.1(2) tells us that $w0 \in X$. And, by Lemma PS4.3.2(2), we have that $\mathbf{dsfxs}(w0) = (\mathbf{dsfxs} w)^- \cup \{0\} = \{-1, 0\}^- \cup \{0\} = \{-2, -1\} \cup \{0\} = \{-2, -1, 0\}$.

(B, 1 \rightarrow C) Suppose $w \in \Lambda_B$, and assume the inductive hypothesis: $w \in X$ and $\mathbf{dsfxs} w = \{-1, 0\}$. We must show that $w1 \in X$ and $\mathbf{dsfxs}(w1) = \{0, 1\}$. Since $w \in X$ and $2 \notin \mathbf{dsfxs} w$, Lemma PS4.3.1(3) tells us that $w1 \in X$. And, by Lemma PS4.3.2(3), we have that $\mathbf{dsfxs}(w1) = (\mathbf{dsfxs} w)^+ \cup \{0\} = \{-1, 0\}^+ \cup \{0\} = \{0, 1\} \cup \{0\} = \{0, 1\}$.

(C, 0 \rightarrow B) Suppose $w \in \Lambda_C$, and assume the inductive hypothesis: $w \in X$ and $\mathbf{dsfxs} w = \{0, 1\}$. We must show that $w0 \in X$ and $\mathbf{dsfxs}(w0) = \{-1, 0\}$. Since $w \in X$ and $-2 \notin \mathbf{dsfxs} w$, Lemma PS4.3.1(2) tells us that $w0 \in X$. And, by Lemma PS4.3.2(2), we have that $\mathbf{dsfxs}(w0) = (\mathbf{dsfxs} w)^- \cup \{0\} = \{0, 1\}^- \cup \{0\} = \{-1, 0\} \cup \{0\} = \{-1, 0\}$.

- (C,1 \rightarrow E) Suppose $w \in \Lambda_C$, and assume the inductive hypothesis: $w \in X$ and $\mathbf{dsfxs} w = \{0,1\}$. We must show that $w1 \in X$ and $\mathbf{dsfxs}(w1) = \{0,1,2\}$. Since $w \in X$ and $2 \notin \mathbf{dsfxs} w$, Lemma PS4.3.1(3) tells us that $w1 \in X$. And, by Lemma PS4.3.2(3), we have that $\mathbf{dsfxs}(w1) = (\mathbf{dsfxs} w)^+ \cup \{0\} = \{0,1\}^+ \cup \{0\} = \{1,2\} \cup \{0\} = \{0,1,2\}$.
- (D,0 \rightarrow G) Suppose $w \in \Lambda_D$, and assume the inductive hypothesis: $w \in X$ and $\mathbf{dsfxs} w = \{-2,-1,0\}$. We must show that $w0 \notin X$. Since $-2 \in \mathbf{dsfxs} w$, there is a suffix v of w such that $\mathbf{diff} v = -2$. Thus $v0$ is a suffix of $w0$, and $\mathbf{diff}(v0) = \mathbf{diff} v + \mathbf{diff} 0 = -2 + -1 = -3$. Since $v0$ is a substring of $w0$ and $\mathbf{diff}(v0) = -3$, it follows that $w0 \notin X$.
- (D,1 \rightarrow F) Suppose $w \in \Lambda_D$, and assume the inductive hypothesis: $w \in X$ and $\mathbf{dsfxs} w = \{-2,-1,0\}$. We must show that $w1 \in X$ and $\mathbf{dsfxs}(w1) = \{-1,0,1\}$. Since $w \in X$ and $2 \notin \mathbf{dsfxs} w$, Lemma PS4.3.1(3) tells us that $w1 \in X$. And, by Lemma PS4.3.2(3), we have that $\mathbf{dsfxs}(w1) = (\mathbf{dsfxs} w)^+ \cup \{0\} = \{-2,-1,0\}^+ \cup \{0\} = \{-1,0,1\} \cup \{0\} = \{-1,0,1\}$.
- (E,0 \rightarrow F) Suppose $w \in \Lambda_E$, and assume the inductive hypothesis: $w \in X$ and $\mathbf{dsfxs} w = \{0,1,2\}$. We must show that $w0 \in X$ and $\mathbf{dsfxs}(w0) = \{-1,0,1\}$. Since $w \in X$ and $-2 \notin \mathbf{dsfxs} w$, Lemma PS4.3.1(2) tells us that $w0 \in X$. And, by Lemma PS4.3.2(2), we have that $\mathbf{dsfxs}(w0) = (\mathbf{dsfxs} w)^- \cup \{0\} = \{0,1,2\}^- \cup \{0\} = \{-1,0,1\} \cup \{0\} = \{-1,0,1\}$.
- (E,1 \rightarrow G) Suppose $w \in \Lambda_E$, and assume the inductive hypothesis: $w \in X$ and $\mathbf{dsfxs} w = \{0,1,2\}$. We must show that $w1 \notin X$. Since $2 \in \mathbf{dsfxs} w$, there is a suffix v of w such that $\mathbf{diff} v = 2$. Thus $v1$ is a suffix of $w1$, and $\mathbf{diff}(v1) = \mathbf{diff} v + \mathbf{diff} 1 = 2 + 1 = 3$. Since $v1$ is a substring of $w1$ and $\mathbf{diff}(v1) = 3$, it follows that $w1 \notin X$.
- (F,0 \rightarrow D) Suppose $w \in \Lambda_F$, and assume the inductive hypothesis: $w \in X$ and $\mathbf{dsfxs} w = \{-1,0,1\}$. We must show that $w0 \in X$ and $\mathbf{dsfxs}(w0) = \{-2,-1,0\}$. Since $w \in X$ and $-2 \notin \mathbf{dsfxs} w$, Lemma PS4.3.1(2) tells us that $w0 \in X$. And, by Lemma PS4.3.2(2), we have that $\mathbf{dsfxs}(w0) = (\mathbf{dsfxs} w)^- \cup \{0\} = \{-1,0,1\}^- \cup \{0\} = \{-2,-1,0\} \cup \{0\} = \{-2,-1,0\}$.
- (F,1 \rightarrow E) Suppose $w \in \Lambda_F$, and assume the inductive hypothesis: $w \in X$ and $\mathbf{dsfxs} w = \{-1,0,1\}$. We must show that $w1 \in X$ and $\mathbf{dsfxs}(w1) = \{0,1,2\}$. Since $w \in X$ and $2 \notin \mathbf{dsfxs} w$, Lemma PS4.3.1(3) tells us that $w1 \in X$. And, by Lemma PS4.3.2(3), we have that $\mathbf{dsfxs}(w1) = (\mathbf{dsfxs} w)^+ \cup \{0\} = \{-1,0,1\}^+ \cup \{0\} = \{0,1,2\} \cup \{0\} = \{0,1,2\}$.
- (G,0 \rightarrow G) Suppose $w \in \Lambda_G$, and assume the inductive hypothesis: $w \notin X$. We must show that $w0 \notin X$. Since $w \notin X$ but $w \in \Lambda_G \subseteq (\mathbf{alphabet} M)^* = \{0,1\}^*$, it follows that there is a substring v of w such $-2 \leq \mathbf{diff} v \leq 2$ is false. But v is also a substring of $w0$, and so $w0 \notin X$.
- (G,1 \rightarrow G) Suppose $w \in \Lambda_G$, and assume the inductive hypothesis: $w \notin X$. We must show that $w1 \notin X$. Since $w \notin X$ but $w \in \Lambda_G \subseteq (\mathbf{alphabet} M)^* = \{0,1\}^*$, it follows that there is a substring v of w such $-2 \leq \mathbf{diff} v \leq 2$ is false. But v is also a substring of $w1$, and so $w1 \notin X$.

□

Proposition PS4.3.4

$L(M) = X$.

Proof. We show that $L(M) \subseteq X \subseteq L(M)$.

$(L(M) \subseteq X)$ Suppose $w \in L(M)$. Because $A_M = \{A, B, C, D, E, F\}$, we have that $w \in L(M) = \Lambda_A \cup \Lambda_B \cup \Lambda_C \cup \Lambda_D \cup \Lambda_E \cup \Lambda_F$. Thus, by Lemma PS4.3.3(A–F), we have that $w \in X$.

$(X \subseteq L(M))$ Suppose $w \in X$. Since $X \subseteq \{0, 1\}^*$, we have that $w \in \{0, 1\}^*$. Suppose, toward a contradiction, that $w \notin L(M)$. Because $w \notin L(M) = \Lambda_A \cup \Lambda_B \cup \Lambda_C \cup \Lambda_D \cup \Lambda_E \cup \Lambda_F$ and $w \in \{0, 1\}^* = (\mathbf{alphabet} M)^* = \Lambda_A \cup \Lambda_B \cup \Lambda_C \cup \Lambda_D \cup \Lambda_E \cup \Lambda_F \cup \Lambda_G$, we must have that $w \in \Lambda_G$. But then Lemma PS4.3.3(G) tells us that $w \notin X$ —contradiction. Thus $w \in L(M)$.

□