CS 591 S2—Formal Language Theory: Integrating Experimentation and Proof—Fall 2018

Final Examination

Wednesday, December 19, 12:30–2:30pm

Question 1 (10 points)

Let **RegLan**, **CFLan**, **RecLan** and **RELan** be, as usual, the sets of all regular, context-free, recursive and recursively enumerable languages. Fill in each cell of the following table with a "Y" (for "yes") or "N" (for "no") to indicate what closure properties these four sets of languages have:

	union	concatenation	closure	intersection	difference
RegLan					
CFLan					
RecLan					
RELan					

E.g., at the intersection of the row **CFLan** and column "difference", put a "Y" if you think the context-free languages are closed under set difference ($L_1 - L_2$), and a "N" if you think the context-free languages are not closed under set difference.

Question 2 (20 points)

Let

$$X = \{\, \mathsf{0}^i\mathsf{1}^j\mathsf{2}^k \mid i,j,k \in \mathbb{N} \text{ and } (i = j \text{ or } j = k)\,\}.$$

Find a grammar G such that $L(G) = X$.

Question 3 (10 points)

Disprove the following statement:

For all languages A and B,

$$(A \cap B)^* = A^* \cap B^*.$$

(E.g., find languages A and B so the equation doesn't hold, and show it doesn't hold.)

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Question 4 (15 points)

Suppose x is a string, Y is a set of strings, and $x \notin Y$. Suppose we have a DFA M such that $L(M) = Y \cup \{x\}$. Explain how we can turn M into a DFA N such that $L(N) = Y$. Explain why your answer is correct.

Question 5 (30 points)

Let

$$X = \{\, w \in \{\mathsf{0},\mathsf{1}\}^* \mid \mathsf{00} \text{ is not a substring of } w \,\}.$$

Thus, for all $w \in \{\mathsf{0},\mathsf{1}\}^*$:

- $w \in X$ iff $\mathsf{00}$ is not a substring of w;
- $w \notin X$ iff $\mathsf{00}$ is a substring of w.

Find a DFA M such that $L(M) = X$, and—using our standard approach, involving induction on Λ—prove that your answer is correct.

In your proof, you may use the parts of the following lemma (you *don't* have to prove it):

Lemma 5.1
(1) $\% \in X$.

(2) For all $w \in \{\mathsf{0},\mathsf{1}\}^*$, *if* $w \in X$, *then* $w\mathsf{1} \in X$.

(3) For all $w \in \{\mathsf{0},\mathsf{1}\}^*$, *if* $w \in X$ *and* $\mathsf{0}$ *is not a suffix of* w, *then* $w\mathsf{0} \in X$.

(4) For all $w \in \{\mathsf{0},\mathsf{1}\}^*$, *if* $\mathsf{0}$ *is a suffix of* w, *then* $w\mathsf{0} \notin X$.

(5) For all $w \in \{\mathsf{0},\mathsf{1}\}^*$ *and* $a \in \{\mathsf{0},\mathsf{1}\}$, *if* $w \notin X$, *then* $wa \notin X$.

Question 6 (15 points)

Let

$$X = \{\, \mathsf{0}^i\mathsf{1}^j\mathsf{2}^k\mathsf{3}^l \mid i,j,k,l \in \mathbb{N} \text{ and } i < l \text{ and } j > k \text{ and } i+j \text{ is even and } k+l \text{ is odd } \}.$$

Prove that X is not regular.