COMP 150-EFL—Experimenting with Formal Languages—Spring 2012

Problem Set 1

Due by 4:30 p.m. on Monday, February 6

Problem 1 (25 points)

Solve Exercise 1.1.7, using the Schröder-Bernstein Theorem to show that $\mathbb{N} \cong \mathbb{N} \times \mathbb{N}$. Hint: use the following consequence of the Fundamental Theorem of Arithmetic: if two finite, ascending (each element is $\leq$ the next) sequences of prime numbers (natural numbers that are at least 2 and have no divisors other than 1 and themselves) have the same product (the product of the empty sequence is 1), then they are equal.

Problem 2 (25 points)

Use strong induction to prove that, for all $n \in \mathbb{N}$, if $n \geq 18$, then there are $i, j \in \mathbb{N}$ such that $n = 4i + 7j$.

Problem 3 (25 points)

Suppose R is a well-founded relation on a set A. Define a relation S on $\textbf{List}\, A$ by: S is the set of all (xs, ys) such that $xs, ys \in \textbf{List}\, A$ and either:

- $|xs| < |ys|$; or
- $|xs| = |ys|$ and, there is an $i \in [1 : |xs|]$ such that
  - for all $j \in [1 : i - 1]$, $xs\, j = ys\, j$, and
  - $xs\, i\ R\ ys\, i$.

Prove that S is well-founded on $\textbf{List}\, A$.

Problem 4 (25 points)

Let X be a set, and suppose Y is a nonempty subset of $\textbf{Tree}\, X$. Use the principle of induction on trees to prove that, for all $tr \in \textbf{Tree}\, X$, if $tr \in Y$, then Y has a $\textbf{pred}_{\textbf{Tree}\, X}$-minimal element. (See Proposition 1.3.5.)