

Problem Set 3

Due by 4:30 p.m. on Wednesday, February 29

Problem 1 (20 points)

Define a function $\mathbf{diff} \in \{0, 1\}^* \rightarrow \mathbb{Z}$ by: for all $w \in \{0, 1\}^*$,

$\mathbf{diff} w =$ the number of 1's in w – the number of 0's in w .

Let $X = \{w \in \{0, 1\}^* \mid |w| \leq 6 \text{ and } \mathbf{diff} w = 0\}$.

Use Forlan to find and show the correctness of a regular expression α such that $L(\alpha) = X$. Try to make α as simple as possible (see `Reg.compareComplexity`), and use Forlan to display the size, number of concatenations, and number of symbols of α , as well as whether α is standardized. Try to do as much as possible of the work of finding and showing the correctness of α using Forlan. (Include a listing of your Forlan session.)

Problem 2 (15 points)

Use `Reg.locallySimplifyTrace` to illustrate how reduction rule (20) works. (Include a listing of your Forlan session.)

Problem 3 (65 points)

Let \mathbf{diff} be as in Problem 1, and define $Y = \{w \in \{0, 1\}^* \mid \text{for all prefixes } v \text{ of } w, -2 \leq \mathbf{diff} v \leq 2\}$.

(a) Find a regular expression α such that $L(\alpha) = Y$. [15 points]

(b) Prove that your answer to part (a) is correct. [50 points]