CS 591 S2—Formal Language Theory: Integrating Experimentation and Proof—Fall 2019

Problem Set 4

Due by 12:30pm on Friday, November 1

Problem 1 (20 points)

Define a function **diff** $\in \{0,1\}^* \to \mathbb{Z}$ by: for all $w \in \{0,1\}^*$,

$$\textbf{diff}\ w = \text{the number of 1's in } w - \text{the number of 0's in } w.$$

Let $X = \{\, w \in \{0,1\}^* \mid \text{for all substrings } v \text{ of } w,\ -2 \leq \textbf{diff}\ v \leq 2 \,\}$. Find a DFA M such that $L(M) = X$.

Problem 2 (20 points)

Define a Forlan/SML function

```
val test = fn : int -> dfa -> str option * str option
```

such that, for all $n \in \mathbb{N}$, `test` n returns a function f such that, for all DFAs N, $f\ N$ returns a pair $(xOpt, yOpt)$ such that:

- If there is an element of $\{0,1\}^*$ of length no more than n that is in X but is not accepted by N, then $xOpt =$ `SOME` x for some such x; otherwise, $xOpt =$ `NONE`.
- If there is an element of $\{0,1\}^*$ of length no more than n that is not in X but is accepted by N, then $yOpt =$ `SOME` y for some such y; otherwise, $yOpt =$ `NONE`.

Use `test` to test your DFA M on all elements of $\{0,1\}^*$ of length no more than 15. Put your definition of `test` in a file `ps4-p2.sml`, and email this file to me. (Include a transcript of your Forlan session.)

Problem 3 (60 points)

Use our standard method of proving the correctness of DFAs—involving induction on Λ and proof by contradiction—to prove that $L(M) = X$.