

CS 591 S2—Formal Language Theory: Integrating Experimentation and Proof—Fall 2019

Problem Set 5

Due by 12:30pm on Friday, November 15

Problem 1 (20 points)

Solve Exercise 3.13.11 of the book: Suppose N' and N are equivalent DFAs with alphabet Σ , all of whose states are reachable (for all $q \in Q_{N'}$, there is a $w \in \Sigma^*$ such that $\delta_{N'}(s_{N'}, w) = q$; and for all $q \in Q_N$, there is a $w \in \Sigma^*$ such that $\delta_N(s_N, w) = q$). Suppose h is a bijection from $Q_{N'}$ to Q_N such that, for all $w \in \Sigma^*$, $h(\delta_{N'}(s_{N'}, w)) = \delta_N(s_N, w)$. Prove that h is an isomorphism from N' to N .

Problem 2 (80 points)

Define a function **DCS** (for “disjoint, correlated substrings”) from $\{0, 1\}^* \times \{0, 1\}^*$ to $\mathcal{P}(\{0, 1\}^*)$ by: for all $x, y \in \{0, 1\}^*$, **DCS**(x, y) is the set of all $w \in \{0, 1\}^*$ such that:

- for all $u, v \in \{0, 1\}^*$, if $w = uxv$, then y is a substring of u or v ; and
- for all $u, v \in \{0, 1\}^*$, if $w = uyv$, then x is a substring of u or v .

(a) Use the functions/algorithms given in the book to define a function/algorithm **dcsDFA** $\in \{0, 1\}^* \times \{0, 1\}^* \rightarrow \mathbf{DFA}$ such that, for all $x, y \in \{0, 1\}^*$:

- $L(\mathbf{dcsDFA}(x, y)) = \mathbf{DCS}(x, y)$; and
- $\mathbf{minimize}(\mathbf{dcsDFA}(x, y))$ is isomorphic to $\mathbf{dcsDFA}(x, y)$.

[25 points]

(b) In the file `ps5.sm1`, define a Forlan/SML function

```
val dcsDFA : str * str -> dfa
```

that implements your definition of **dcsDFA**. You should assume that **dcsDFA** will only be called with elements of $\{0, 1\}^*$.

Systematically test your definition of **dcsDFA**. (Include a transcript of your Forlan session.) Email to me copies of your `ps5.sm1` as well as the file containing your testing code, in addition to including listings of them as part of your solution to the problem set. [25 points]

(c) Prove that your definition of **dcsDFA** is correct.

[30 points]