**COMP 150-EFL—Experimenting with Formal Languages—Spring 2012**

# Problem Set 7

**Due by 4:30 p.m. on Wednesday, May 2**

## Problem 1 (50 points)

Let $X = \{\, w \in \{\mathsf{0}\}^* \mid |w| \text{ is prime} \,\}$. Either prove or disprove the following statement:

$X$ is context-free.

## Problem 2 (50 points)

Define functions $\mathbf{zeros}, \mathbf{ones} \in \{\mathsf{0},\mathsf{1}\}^*$ by: for all $w \in \{\mathsf{0},\mathsf{1}\}^*$,

$$\mathbf{zeros}\, w = \text{the number of occurrences of } \mathsf{0} \text{ in } w, \text{ and}$$
$$\mathbf{ones}\, w = \text{the number of occurrences of } \mathsf{1} \text{ in } w.$$

Let $X$ be the set of all $w \in \{\mathsf{0},\mathsf{1}\}^*$ such that either:

- $\mathbf{zeros}\, w = 2(\mathbf{ones}\, w)$ and $\mathbf{ones}\, w$ is even; or
- $\mathbf{ones}\, w = 2(\mathbf{zeros}\, w)$ and $\mathbf{zeros}\, w$ is even.

Use Forlan to help find a simplified grammar $G$ such that $L(G) = X$. Don't worry about making $G$ elegant or easy to understand. Informally explain why $G$ generates $X$. And use Forlan to systematically test the correctness of $G$.