CS 591 S2—Formal Language Theory: Integrating Experimentation and Proof—Fall 2018

Problem Set 1

Due by 12:30pm on Thursday, September 20

You must submit your problem set solution as a hard copy, either: at the beginning of class; or, no later than 12:05pm, via the CS Department drop box labeled "CS 591 S2".

Problem 1 (20 points)

Suppose X is a set, and $x \in X$. What are the elements of $\emptyset \to X$, $X \to \emptyset$, $\{x\} \to X$ and $X \to \{x\}$? Prove that your answers are correct.

Problem 2 (20 points)

Use mathematical induction (Theorem 1.2.1) to prove that, for all $n \in \mathbb{N}$, if $n \geq 4$, then $2^n < n!$.

Problem 3 (20 points)

Define $f \in \mathbb{N} \to \mathbb{N}$ by:

$$f\,n = \begin{cases} n/2, & \text{if } n \text{ is even,} \\ 0, & \text{if } n = 1, \\ n+1, & \text{if } n > 1 \text{ and } n \text{ is odd.} \end{cases}$$

Given $n \in \mathbb{N}$, define the l-th *iteration of f on n* ($f^l(n) \in \mathbb{N}$), for $l \in \mathbb{N}$, by recursion:

$$f^0(n) = n,$$
$$f^{l+1}(n) = f(f^l(n)).$$

Then we have that:

(1) For all $n \in \mathbb{N}$, $f^1(n) = f\,n$.

(2) For all $l, m, n \in \mathbb{N}$, $f^{l+m}(n) = f^m(f^l(n))$.

Use strong induction to show that, for all $n \in \mathbb{N}$, there is an $l \in \mathbb{N}$ such that $f^l(n) = 0$.

Problem 4 (20 points)

Let $A = \{0, 1\}$ and $R = \{(0, 1), (1, 0)\}$, so that R is a relation on A. Suppose we were allowed to use Theorem 1.2.8 (Principle of Well-founded Induction) with R (even though it is not well-founded). Prove an obviously false statement.

Problem 5 (20 points)

Recall the following definition from Section 1.3: Given a set X, we say that a set U is X-*closed* iff, for all $x \in X$ and $trs \in \mathbf{List}\, U$, $(x, trs) \in U$.

Either prove or disprove the following statement:

> For all sets X and nonempty sets of X-closed sets $\mathcal{W}$, $\bigcup \mathcal{W}$ is an X-closed set.