CS 591 S2—Formal Language Theory: Integrating Experimentation and Proof—Fall 2018

Problem Set 3

Due by 12:30pm on Thursday, October 18

You must submit your problem set solution as a hard copy, either: at the beginning of class; or, no later than 12:05pm, via the CS Department drop box labeled "CS 591 S2".

Problem 1 (20 Points)

Let $X = \{0,1\}^4 - \{\mathsf{0101}, \mathsf{1010}\}$. Use Forlan to find and show the correctness of a regular expression α such that $L(\alpha) = X$. Try to make α as simple as possible (see `Reg.compareComplexity`), and use Forlan to display the closure complexity, size, number of concatenations, and number of symbols of α, as well as whether α is standardized. Try to do as much as possible of the work of finding and showing the correctness of α using Forlan. (Include a listing of your Forlan session.)

Problem 2 (35 Points)

Consider reduction rule (14) from Section 3.3.3 of the book:

> If $\mathbf{not}(\mathbf{hasEmp}\,\alpha)$ and $\mathbf{cc}\,\alpha \cup \overline{\mathbf{cc}\,\beta} <_{cc} \overline{\overline{\mathbf{cc}\,\beta}}$,
> then $(\alpha\beta^*)^* \to \% + \alpha(\alpha+\beta)^*$.

(a) What is the rationale for the rule only being applicable when $\mathbf{not}(\mathbf{hasEmp}\,\alpha)$? [5 points]

(b) Prove that, for all $\alpha, \beta \in \mathbf{Reg}$, if $\mathbf{cc}\,\alpha \cup \overline{\mathbf{cc}\,\beta} <_{cc} \overline{\overline{\mathbf{cc}\,\beta}}$, then $\mathbf{cc}(\% + \alpha(\alpha+\beta)^*) <_{cc} \mathbf{cc}((\alpha\beta^*)^*)$. [10 points]

(c) Prove that, for all languages A and B,

$$(AB^*)^* = \{\%\} \cup A(A \cup B)^*.$$

[15 points]

(d) Prove that, for all $\alpha, \beta \in \mathbf{Reg}$, $(\alpha\beta^*)^* \approx \% + \alpha(\alpha+\beta)^*$. [5 points]

Problem 3 (45 points)

Let $X = \{\, w \in \{\mathtt{0}, \mathtt{1}\}^* \mid \mathtt{010} \text{ is not a substring of } w \,\}$.

(a) Find a regular expression α such that $L(\alpha) = X$. [10 points]

(b) Prove that your answer to Part (a) is correct. [35 points]