

CS 591 S2—Formal Language Theory: Integrating Experimentation and Proof—Fall 2018

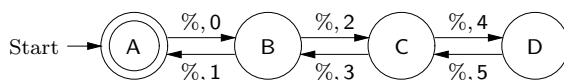
Problem Set 4

Due by 12:30pm on Thursday, November 1

You must submit your problem set solution as a hard copy, either: at the beginning of class; or, no later than 12:05pm, via the CS Department drop box labeled “CS 591 S2”. In addition, see the instructions in Problem 2 for emailing the Forlan code for Problem 2(b) to me, no later than 12:30pm.

Problem 1 (25 points)

Let M be the EFA



(a) Use Forlan to find and display minimum-length labeled paths explaining why the strings 012345012345 and 04320123552 are accepted by M . Include a transcript of your Forlan session. [10 points]

(b) Give a step-by-step explanation of how our EFA-to-NFA conversion algorithm converts M into an NFA, N . Draw N , and use Forlan to check that your answer is correct. Include a transcript of your Forlan session. [10 points]

(c) Describe the language $L(M)$ as simply as possible, without mentioning M or N . [5 points]

Problem 2 (75 points)

Define $f \in \{0, 1\}^* \rightarrow \mathcal{P}(\{0, 1\}^*)$ by:

$$f w = \{ y \in \{0, 1\}^* \mid y000 \text{ is a prefix of } w \}.$$

Since $f(w)$ is always finite, we can define $g \in \{0, 1\}^* \rightarrow \mathbb{N}$ by: $g w = |f w|$. Let $X = \{ w \in \{0, 1\}^* \mid g w \text{ is even} \}$.

For example:

- $f(0001000) = \{ \% , 0001 \}$, so that $g(0001000) = | \{ \% , 0001 \} | = 2$, and thus $0001000 \in X$; and

- $f(00001000) = \{\%, 0, 00001\}$, so that $g(00001000) = |\{\%, 0, 00001\}| = 3$, and thus $00001000 \notin X$.

(Informally, if $w \in \{0, 1\}^*$, then $g w$ is the *number of occurrences of 000 in w* . But in part (c), you must work in terms of the definitions of f and g , proving whatever properties you need, in a lemma or lemmas.)

(a) Find a DFA M such that $L(M) = X$. [15 points]

(b) In a file `ps4-p2-dfa.txt`, express M in Forlan's syntax. Write a Forlan program `ps4-p2-testing.sml` defining a function

```
val test = fn : dfa -> bool
```

that thoroughly tests a solution to part (a), returning `true`, if it passes all tests, and `false`, otherwise.

Include printouts of `ps4-p2-dfa.txt` and `ps4-p2-testing.sml` in your hard copy submission, as well as a transcript of the following Forlan session:

```
- use "ps4-p2-testing.sml";
[opening ps4-p2-testing.sml]
...
val test = fn : dfa -> bool
val it = () : unit
- val dfa = DFA.input "ps4-p2-dfa.txt";
val dfa = - : dfa
- test dfa;
val it = true : bool
```

Also email `ps4-p2-dfa.txt` and `ps4-p2-testing.sml` as plain text attachments to me (`stough@bu.edu`), with a subject line including “[591S2:PS4]”. [15 points]

(c) Use our standard technique for proving DFAs correct (induction on Λ , plus proof by contradiction) to prove that $L(M) = X$. [45 points]