CS 591 S2—Formal Language Theory: Integrating Experimentation and Proof—Fall 2018

Problem Set 7

Due by 12:30pm on Thursday, December 13

You must submit your problem set solution as a hard copy by 12:30pm via the CS Department drop box labeled "CS 591 S2". In addition, see the instructions in Problem 3 for emailing the grammar of Problem 3 to me, no later than 12:30pm.

Problem 1 (20 points)

In Section 4.8 of the book, we studied algorithms for converting regular expressions and finite automata to grammars. In Forlan, these algorithms are implemented as the functions

```
val regToGram : reg -> gram
val faToGram  : fa -> gram
```

As is mentioned in that section, because we can convert back and forth between regular expressions and finite automata, we could have implemented only one of the above functions, defining the other in terms of it.

Define Forlan functions

```
val regToGram' : reg -> gram
val faToGram'  : fa -> gram
```

where:

- `regToGram'` converts a regular expression to a grammar, and is implemented using `faToGram`; and
- `faToGram'` converts a finite automaton to a grammar, and is implemented using `regToGram`.

Show how the outputs of your functions compare to the outputs of the originals on an example regular expression and an example finite automaton. Before displaying the resulting grammars, rename their variables canonically. Include a transcript of your Forlan session. (Nothing from this problem should be emailed to me.)

Problem 2 (25 points)

Let Σ be the alphabet $\{0, 1, 2, 3\}$. Let **zeros**, **ones**, **twos** and **threes** be the functions from Σ^* to $\mathbb{N}$ defined by:

$$\begin{aligned}
\textbf{zeros}\, w &= \text{the number of occurrences of } 0 \text{ in } w, \\
\textbf{ones}\, w &= \text{the number of occurrences of } 1 \text{ in } w, \\
\textbf{twos}\, w &= \text{the number of occurrences of } 2 \text{ in } w, \\
\textbf{threes}\, w &= \text{the number of occurrences of } 3 \text{ in } w.
\end{aligned}$$

Define a language X by:

$$X = \{\, w \in \Sigma^* \mid \textbf{zeros}\, w = \textbf{ones}\, w \text{ and } \textbf{twos}\, w = \textbf{threes}\, w \,\}.$$

Prove that X is not context-free.

Problem 3 (55 points)

Define a language X by:

$$X = \{\, 0^i 1^j 2^k 3^l \mid i, j, k, l \in \mathbb{N} \text{ and } i < l \text{ and } j > k \text{ and } i + j \text{ is even and } k + l \text{ is odd } \}.$$

(a) Use Forlan to help find a grammar G such that $L(G) = X$.

- Try to do as much as possible of the work of finding G using Forlan.
- But as a secondary goal, attempt to minimize the number of productions in G.

Rename the variables of your grammar canonically. Output G into the file `ps7-p3-gram.txt`. Include a transcript of your Forlan session. Also email `ps7-p3-gram.txt` as a plain text attachment to me (`stough@bu.edu`), with a subject line including "`[591S2:PS7]`. [40 points]

(b) Use Forlan to generate the set of all elements of X of length 25. Display the size of this set, as well as its elements. Include a transcript of your Forlan session. (Hint: make use of the work you did in part (a). You can get partial credit by solving the problem for a smaller odd number than 25.) [15 points]