CIS 705 — Programming Languages — Spring 2010

Assignment 1 (100 Points)

Due by 2:30 p.m. on Thursday, February 11

The context for this assignment is Chapters 2-3 of *TAPL*.

Terms

The set of *terms* is inductively defined by:

t ::=	terms:
true	constant true
false	constant false
if t then t else t	conditional
0	constant zero
succ t	successor
pred t	predecessor
iszero t	zero test
error	constant error
t otherwise t	error recovery

We treat true, false, if • then • else, 0, succ •, pred •, iszero •, error and • otherwise • as constructors.

The *inversion lemma for terms* says that, for all terms s, either:

(True) $s =$ true; or

(False) $s =$ false; or

(Conditional) $s =$ if t_1 then t_2 else t_3, for some terms t_1, t_2 and t_3; or

(Zero) $s = 0$; or

(Successor) $s =$ succ t, for some term t; or

(Predecessor) $s =$ pred t, for some term t; or

(Zero Test) $s =$ iszero t, for some term t; or

(Error) $s =$ error; or

(Error Recovery) $s = t_1$ otherwise t_2, for some terms t_1 and t_2.

Suppose P is a predicate on terms. The *principle of structural induction on terms* says that,

$$\text{for all terms } t, P(t)$$

follows from showing:

(True) $P(\mathsf{true})$;

(False) $P(\mathsf{false})$;

(Conditional) for all terms t_1, t_2 and t_3, if (†) $P(t_1)$, $P(t_2)$ and $P(t_3)$, then $P(\mathsf{if}\ t_1\ \mathsf{then}\ t_2\ \mathsf{else}\ t_3)$;

(Zero) $P(0)$;

(Successor) for all terms t, if (†) $P(t)$, then $P(\mathsf{succ}\ t)$;

(Predecessor) for all terms t, if (†) $P(t)$, then $P(\mathsf{pred}\ t)$;

(Zero Test) for all terms t, if (†) $P(t)$, then $P(\mathsf{iszero}\ t)$;

(Error) $P(\mathsf{error})$;

(Error Recovery) for all terms t_1 and t_2, if (†) $P(t_1)$ and $P(t_2)$, then $P(t_1\ \mathsf{otherwise}\ t_2)$.

We refer to (†) as the *inductive hypothesis*.

Values and Answers

The set of *boolean values* is defined by:

bv ::=	boolean values:
true	true boolean value
false	false boolean value

The set of *numeric values* is inductively defined by:

nv ::=	numeric values:
0	zero numeric value
succ nv	successor numeric value

The *inversion lemma for numeric values* says that, for all numeric values nv, either:

(Zero) $nv = \mathsf{0}$; or

(Successor) $nv = \mathsf{succ}\ nv'$, for some numeric value nv'.

Suppose P is a predicate on numeric values. The *principle of structural induction on numeric values* says that,

$$\text{for all numeric values } nv, P(nv)$$

follows from showing:

(Zero) $P(\mathsf{0})$;

(Successor) for all numeric values nv, if (†) $P(nv)$, then $P(\mathsf{succ}\ nv)$.

We refer to (†) as the *inductive hypothesis*.

The set of *values* is defined by:

$v ::=$	values:
bv	boolean value
nv	numeric value

The set of *answers* is defined by:

$a ::=$	answers:
v	value answer
error	error answer

It is easy to prove that every answer is a term.

Evaluation Relation

The *evaluation relation* $\boxed{t \to t'}$ on terms is inductively defined by:

$$\mathsf{if\ true\ then}\ t_2\ \mathsf{else}\ t_3 \to t_2 \qquad \text{(E-IfTrue)}$$

$$\mathsf{if\ false\ then}\ t_2\ \mathsf{else}\ t_3 \to t_3 \qquad \text{(E-IfFalse)}$$

$$\mathsf{if}\ nv\ \mathsf{then}\ t_2\ \mathsf{else}\ t_3 \to \mathsf{error} \qquad \text{(E-IfNum)}$$

$$\mathsf{if\ error\ then}\ t_2\ \mathsf{else}\ t_3 \to \mathsf{error} \qquad \text{(E-IfError)}$$

$$\frac{t_1 \to t_1'}{\mathsf{if}\ t_1\ \mathsf{then}\ t_2\ \mathsf{else}\ t_3 \to \mathsf{if}\ t_1'\ \mathsf{then}\ t_2\ \mathsf{else}\ t_3} \qquad \text{(E-If)}$$

$\mathsf{succ}\ bv \to \mathsf{error}$ (E-SuccBool)

$\mathsf{succ\ error} \to \mathsf{error}$ (E-SuccError)

$$\frac{t_1 \to t_1'}{\mathsf{succ}\ t_1 \to \mathsf{succ}\ t_1'} \quad \text{(E-Succ)}$$

$\mathsf{pred}\ bv \to \mathsf{error}$ (E-PredBool)

$\mathsf{pred}\ 0 \to \mathsf{error}$ (E-PredZero)

$\mathsf{pred}(\mathsf{succ}\ nv) \to nv$ (E-PredSucc)

$\mathsf{pred\ error} \to \mathsf{error}$ (E-PredError)

$$\frac{t_1 \to t_1'}{\mathsf{pred}\ t_1 \to \mathsf{pred}\ t_1'} \quad \text{(E-Pred)}$$

$\mathsf{iszero}\ bv \to \mathsf{error}$ (E-IszeroBool)

$\mathsf{iszero}\ 0 \to \mathsf{true}$ (E-IszeroZero)

$\mathsf{iszero}(\mathsf{succ}\ nv) \to \mathsf{false}$ (E-IszeroSucc)

$\mathsf{iszero\ error} \to \mathsf{error}$ (E-IszeroError)

$$\frac{t_1 \to t_1'}{\mathsf{iszero}\ t_1 \to \mathsf{iszero}\ t_1'} \quad \text{(E-Iszero)}$$

$v\ \mathsf{otherwise}\ t \to v$ (E-OtherwiseValue)

$\mathsf{error\ otherwise}\ t \to t$ (E-OtherwiseError)

$$\frac{t_1 \to t_1'}{t_1\ \mathsf{otherwise}\ t_2 \to t_1'\ \mathsf{otherwise}\ t_2} \quad \text{(E-Otherwise)}$$

The *inversion lemma for the evaluation relation* says that, for all terms s_1 and s_2, if $s_1 \to s_2$, then either:

(IfTrue) $s_1 = \mathsf{if\ true\ then}\ t_2\ \mathsf{else}\ t_3$ and $s_2 = t_2$, for some terms t_2 and t_3; or

(IfFalse) $s_1 = \mathsf{if\ false\ then}\ t_2\ \mathsf{else}\ t_3$ and $s_2 = t_3$, for some terms t_2 and t_3; or

(IfNum) $s_1 = \mathsf{if}\ nv\ \mathsf{then}\ t_2\ \mathsf{else}\ t_3$ and $s_2 = \mathsf{error}$, for some numeric value nv and terms t_2 and t_3; or

(IfError) $s_1 = \mathsf{if\ error\ then}\ t_2\ \mathsf{else}\ t_3$ and $s_2 = \mathsf{error}$, for some terms t_2 and t_3; or

(If) there are terms t_1, t_1', t_2 and t_3 such that $s_1 = \mathsf{if}\ t_1\ \mathsf{then}\ t_2\ \mathsf{else}\ t_3$, $s_2 = \mathsf{if}\ t_1'\ \mathsf{then}\ t_2\ \mathsf{else}\ t_3$ and $t_1 \to t_1'$; or

(SuccBool) $s_1 = \mathsf{succ}\ bv$ and $s_2 = \mathsf{error}$, for some some boolean value bv; or

(SuccError) $s_1 = \mathsf{succ\ error}$ and $s_2 = \mathsf{error}$; or

(Succ) there are terms t_1 and t_1' such that $s_1 = \mathsf{succ}\ t_1$, $s_2 = \mathsf{succ}\ t_1'$ and $t_1 \to t_1'$; or

(PredBool) $s_1 = \mathsf{pred}\ bv$ and $s_2 = \mathsf{error}$, for some boolean value bv; or

(PredZero) $s_1 = \mathsf{pred\ 0}$ and $s_2 = \mathsf{error}$; or

(PredSucc) $s_1 = \mathsf{pred}(\mathsf{succ}\ nv)$ and $s_2 = nv$, for some numeric value nv; or

(PredError) $s_1 = \mathsf{pred\ error}$ and $s_2 = \mathsf{error}$; or

(Pred) there are terms t_1 and t_1' such that $s_1 = \mathsf{pred}\ t_1$, $s_2 = \mathsf{pred}\ t_1'$ and $t_1 \to t_1'$; or

(IszeroBool) $s_1 = \mathsf{iszero}\ bv$ and $s_2 = \mathsf{error}$, for some boolean value bv; or

(IszeroZero) $s_1 = \mathsf{iszero\ 0}$ and $s_2 = \mathsf{true}$; or

(IszeroSucc) $s_1 = \mathsf{iszero}(\mathsf{succ}\ nv)$ and $s_2 = \mathsf{false}$, for some numeric value nv; or

(IszeroError) $s_1 = \mathsf{iszero\ error}$ and $s_2 = \mathsf{error}$; or

(Iszero) there are terms t_1 and t_1' such that $s_1 = \mathsf{iszero}\ t_1$, $s_2 = \mathsf{iszero}\ t_1'$ and $t_1 \to t_1'$; or

(OtherwiseValue) $s_1 = v\ \mathsf{otherwise}\ t$ and $s_2 = v$, for some term t and value v; or

(OtherwiseError) $s_1 = \mathsf{error\ otherwise}\ t$ and $s_2 = t$, for some term t; or

(Otherwise) there are terms t_1 and t_1' such that $s_1 = t_1\ \mathsf{otherwise}\ t_2$, $s_2 = t_1'\ \mathsf{otherwise}\ t_2$ and $t_1 \to t_1'$.

Suppose P is a binary relation on terms. We sometimes write "$P(t_1, t_2)$" for "$(t_1, t_2) \in P$". The *principle of induction on the evaluation relation* says that,

for all terms t_1 and t_2, if $t_1 \to t_2$, then $P(t_1, t_2)$,

follows from showing:

(IfTrue) for all terms t_2 and t_3, $P(\mathsf{if\ true\ then}\ t_2\ \mathsf{else}\ t_3, t_2)$;

(IfFalse) for all terms t_2 and t_3, $P(\mathsf{if\ false\ then}\ t_2\ \mathsf{else}\ t_3, t_3)$;

(IfNum) for all numeric values nv and terms t_2 and t_3, $P(\mathsf{if}\ nv\ \mathsf{then}\ t_2\ \mathsf{else}\ t_3, \mathsf{error})$;

(IfError) for all terms t_2 and t_3, $P(\mathsf{if\ error\ then}\ t_2\ \mathsf{else}\ t_3, \mathsf{error})$;

(If) for all terms t_1, t_1', t_2 and t_3, if $t_1 \to t_1'$ and (†) $P(t_1, t_1')$, then $P(\mathsf{if}\ t_1\ \mathsf{then}\ t_2\ \mathsf{else}\ t_3, \mathsf{if}\ t_1'\ \mathsf{then}\ t_2\ \mathsf{else}\ t_3)$;

(SuccBool) for all boolean values bv, $P(\mathsf{succ}\ bv, \mathsf{error})$;

(SuccError) $P(\mathsf{succ\ error}, \mathsf{error})$;

(Succ) for all terms t_1 and t_1', if $t_1 \to t_1'$ and (†) $P(t_1, t_1')$, then $P(\mathsf{succ}\ t_1, \mathsf{succ}\ t_1')$;

(PredBool) for all boolean values bv, $P(\mathsf{pred}\ bv, \mathsf{error})$;

(PredZero) $P(\mathsf{pred\ 0}, \mathsf{error})$;

(PredSucc) for all numeric values nv, $P(\mathsf{pred}(\mathsf{succ}\ nv), nv)$;

(PredError) $P(\mathsf{pred\ error}, \mathsf{error})$;

(Pred) for all terms t_1 and t_1', if $t_1 \to t_1'$ and (†) $P(t_1, t_1')$, then $P(\mathsf{pred}\ t_1, \mathsf{pred}\ t_1')$;

(IszeroBool) for all boolean values bv, $P(\mathsf{iszero}\ bv, \mathsf{error})$;

(IszeroZero) $P(\mathsf{iszero\ 0}, \mathsf{true})$;

(IszeroSucc) for all numeric values nv, $P(\mathsf{iszero}(\mathsf{succ}\ nv), \mathsf{false})$;

(IszeroError) $P(\mathsf{iszero\ error}, \mathsf{error})$;

(Iszero) for all terms t_1 and t_1', if $t_1 \to t_1'$ and (†) $P(t_1, t_1')$, then $P(\mathsf{iszero}\ t_1, \mathsf{iszero}\ t_1')$;

(OtherwiseValue) for all terms t and values v, $P(v\ \mathsf{otherwise}\ t, v)$;

(OtherwiseError) for all terms t, $P(\mathsf{error\ otherwise}\ t, t)$;

(Otherwise) for all terms t_1 and t_1', if $t_1 \to t_1'$ and (†) $P(t_1, t_1')$, then $P(t_1\ \mathsf{otherwise}\ t_2, t_1'\ \mathsf{otherwise}\ t_2)$.

We refer to (†) as the *inductive hypothesis*.

Reflexive-Transitive Closure of Evaluation Relation

The *reflexive-transitive closure of the evaluation relation* $\boxed{t \to^* t}$ is inductively defined by:

$$\frac{t_1 \to t_2}{t_1 \to^* t_2} \qquad \text{(RTCE-Eval)}$$

$$t \to^* t \qquad \text{(RTCE-Refl)}$$

$$\frac{t_1 \to^* t_2 \qquad t_2 \to^* t_3}{t_1 \to^* t_3} \qquad \text{(RTCE-Trans)}$$

The *inversion lemma for the reflexive-transitive closure of the evaluation relation* says that, for all terms t_1 and t_2, if $t_1 \to^* t_2$, then either:

(Eval) $t_1 \to t_2$; or

(Refl) $t_1 = t_2$; or

(Trans) there is a term t' such that $t_1 \to^* t'$ and $t' \to^* t_2$.

Suppose P is a binary relation on terms. The *principle of induction on the reflexive-transitive closure of the evaluation relation* says that,

$$\text{for all terms } t_1 \text{ and } t_2, \text{ if } t_1 \to^* t_2, \text{ then } P(t_1, t_2),$$

follows from showing:

(Eval) for all terms t_1 and t_2, if $t_1 \to t_2$, then $P(t_1, t_2)$;

(Refl) for all terms t, $P(t, t)$;

(Trans) for all terms t_1, t_2 and t_3, if $t_1 \to^* t_2$ and $t_2 \to^* t_3$, and (†) $P(t_1, t_2)$ and $P(t_2, t_3)$, then $P(t_1, t_3)$.

We refer to (†) as the *inductive hypothesis*.

Complete Evaluation Relation

The *complete evaluation relation* $\boxed{t \Rightarrow a}$ between terms and answers is inductively defined by:

$$\mathsf{true} \Rightarrow \mathsf{true} \qquad \text{(CE-True)}$$

$$\mathsf{false} \Rightarrow \mathsf{false} \qquad \text{(CE-False)}$$

$$\mathsf{0} \Rightarrow \mathsf{0} \qquad \text{(CE-Zero)}$$

$$\mathsf{error} \Rightarrow \mathsf{error} \qquad \text{(CE-Error)}$$

$$\frac{t_1 \Rightarrow \mathsf{true} \qquad t_2 \Rightarrow a}{\mathsf{if}\ t_1\ \mathsf{then}\ t_2\ \mathsf{else}\ t_3 \Rightarrow a} \qquad \text{(CE-IfTrue)}$$

$$\frac{t_1 \Rightarrow \mathsf{false} \qquad t_3 \Rightarrow a}{\mathsf{if}\ t_1\ \mathsf{then}\ t_2\ \mathsf{else}\ t_3 \Rightarrow a} \qquad \text{(CE-IfFalse)}$$

$$\frac{t_1 \Rightarrow nv}{\mathsf{if}\ t_1\ \mathsf{then}\ t_2\ \mathsf{else}\ t_3 \Rightarrow \mathsf{error}} \qquad \text{(CE-IfNum)}$$

$$\frac{t_1 \Rightarrow \mathsf{error}}{\mathsf{if}\ t_1\ \mathsf{then}\ t_2\ \mathsf{else}\ t_3 \Rightarrow \mathsf{error}} \qquad \text{(CE-IfError)}$$

$$\frac{t \Rightarrow bv}{\mathsf{succ}\ t \Rightarrow \mathsf{error}} \qquad \text{(CE-SuccBool)}$$

$$\frac{t \Rightarrow nv}{\mathsf{succ}\ t \Rightarrow \mathsf{succ}\ nv} \quad \text{(CE-SuccNum)}$$

$$\frac{t \Rightarrow \mathsf{error}}{\mathsf{succ}\ t \Rightarrow \mathsf{error}} \quad \text{(CE-SuccError)}$$

$$\frac{t \Rightarrow bv}{\mathsf{pred}\ t \Rightarrow \mathsf{error}} \quad \text{(CE-PredBool)}$$

$$\frac{t \Rightarrow \mathsf{0}}{\mathsf{pred}\ t \Rightarrow \mathsf{error}} \quad \text{(CE-PredZero)}$$

$$\frac{t \Rightarrow \mathsf{succ}\ nv}{\mathsf{pred}\ t \Rightarrow nv} \quad \text{(CE-PredSucc)}$$

$$\frac{t \Rightarrow \mathsf{error}}{\mathsf{pred}\ t \Rightarrow \mathsf{error}} \quad \text{(CE-PredError)}$$

$$\frac{t \Rightarrow bv}{\mathsf{iszero}\ t \Rightarrow \mathsf{error}} \quad \text{(CE-IszeroBool)}$$

$$\frac{t \Rightarrow \mathsf{0}}{\mathsf{iszero}\ t \Rightarrow \mathsf{true}} \quad \text{(CE-IszeroZero)}$$

$$\frac{t \Rightarrow \mathsf{succ}\ nv}{\mathsf{iszero}\ t \Rightarrow \mathsf{false}} \quad \text{(CE-IszeroSucc)}$$

$$\frac{t \Rightarrow \mathsf{error}}{\mathsf{iszero}\ t \Rightarrow \mathsf{error}} \quad \text{(CE-IszeroError)}$$

$$\frac{t_1 \Rightarrow v}{t_1\ \mathsf{otherwise}\ t_2 \Rightarrow v} \quad \text{(CE-OtherwiseValue)}$$

$$\frac{t_1 \Rightarrow \mathsf{error} \qquad t_2 \Rightarrow a}{t_1\ \mathsf{otherwise}\ t_2 \Rightarrow a} \quad \text{(CE-OtherwiseError)}$$

The *inversion lemma for the complete evaluation relation* says that, for all terms s and answers b, if $s \Rightarrow b$, then either:

(True) $s = \mathsf{true}$ and $b = \mathsf{true}$; or

(False) $s = \mathsf{false}$ and $b = \mathsf{false}$; or

(Zero) $s = \mathsf{0}$ and $b = \mathsf{0}$; or

(Error) $s = \mathsf{error}$ and $b = \mathsf{error}$; or

(IfTrue) there are terms t_1, t_2 and t_3 and an answer a such that $s = \mathsf{if}\ t_1\ \mathsf{then}\ t_2\ \mathsf{else}\ t_3$, $b = a$, $t_1 \Rightarrow \mathsf{true}$ and $t_2 \Rightarrow a$; or

(IfFalse) there are terms t_1, t_2 and t_3 and an answer a such that $s = \mathsf{if}\ t_1\ \mathsf{then}\ t_2\ \mathsf{else}\ t_3$, $b = a$, $t_1 \Rightarrow \mathsf{false}$ and $t_3 \Rightarrow a$; or

(IfNum) there are terms t_1, t_2 and t_3 and a numeric value nv such that $s = \mathsf{if}\ t_1\ \mathsf{then}\ t_2\ \mathsf{else}\ t_3$, $b = \mathsf{error}$ and $t_1 \Rightarrow nv$; or

(IfError) there are terms t_1, t_2 and t_3 such that $s = \mathsf{if}\ t_1\ \mathsf{then}\ t_2\ \mathsf{else}\ t_3$, $b = \mathsf{error}$ and $t_1 \Rightarrow \mathsf{error}$; or

(SuccBool) there is a term t and a boolean value bv such that $s = \mathsf{succ}\ t$, $b = \mathsf{error}$ and $t \Rightarrow bv$; or

(SuccNum) there is a term t and a numeric value nv such that $s = \mathsf{succ}\ t$, $b = \mathsf{succ}\ nv$ and $t \Rightarrow nv$; or

(SuccError) there is a term t such that $s = \mathsf{succ}\ t$, $b = \mathsf{error}$ and $t \Rightarrow \mathsf{error}$; or

(PredBool) there is a term t and a boolean value bv such that $s = \mathsf{pred}\ t$, $b = \mathsf{error}$ and $t \Rightarrow bv$; or

(PredZero) there is a term t such that $s = \mathsf{pred}\ t$, $b = \mathsf{error}$ and $t \Rightarrow \mathsf{0}$; or

(PredSucc) there is a term t and a numeric value nv such that $s = \mathsf{pred}\ t$, $b = nv$ and $t \Rightarrow \mathsf{succ}\ nv$; or

(PredError) there is a term t such that $s = \mathsf{pred}\ t$, $b = \mathsf{error}$ and $t \Rightarrow \mathsf{error}$; or

(IszeroBool) there is a term t and a boolean value bv such that $s = \mathsf{iszero}\ t$, $b = \mathsf{error}$ and $t \Rightarrow bv$; or

(IszeroZero) there is a term t such that $s = \mathsf{iszero}\ t$, $b = \mathsf{true}$ and $t \Rightarrow \mathsf{0}$; or

(IszeroSucc) there is a term t and a numeric value nv such that $s = \mathsf{iszero}\ t$, $b = \mathsf{false}$ and $t \Rightarrow \mathsf{succ}\ nv$; or

(IszeroError) there is a term t such that $s = \mathsf{iszero}\ t$, $b = \mathsf{error}$ and $t \Rightarrow \mathsf{error}$; or

(OtherwiseValue) there are terms t_1 and t_2 and a value v such that $s = t_1\ \mathsf{otherwise}\ t_2$, $b = v$ and $t_1 \Rightarrow v$; or

(OtherwiseError) there are terms t_1 and t_2 and an answer a such that $s = t_1\ \mathsf{otherwise}\ t_2$, $b = a$, $t_1 \Rightarrow \mathsf{error}$ and $t_2 \Rightarrow a$.

Suppose P is a predicate on a term and an answer. The *principle of induction on the complete evaluation relation* says that,

$$\text{for all terms } t \text{ and answers } a, \text{ if } t \Rightarrow a, \text{ then } P(t, a),$$

follows from showing:

(True) $P(\mathsf{true}, \mathsf{true})$;

(False) $P(\mathsf{false}, \mathsf{false})$;

(Zero) $P(\mathsf{0}, \mathsf{0})$;

(Error) $P(\mathsf{error}, \mathsf{error})$;

(IfTrue) for all terms t_1, t_2 and t_3 and answers a, if $t_1 \Rightarrow \mathsf{true}$ and $t_2 \Rightarrow a$, and (†) $P(t_1, \mathsf{true})$ and $P(t_2, a)$, then $P(\mathsf{if}\ t_1\ \mathsf{then}\ t_2\ \mathsf{else}\ t_3, a)$;

(IfFalse) for all terms t_1, t_2 and t_3 and answers a, if $t_1 \Rightarrow \mathsf{false}$ and $t_3 \Rightarrow a$, and (†) $P(t_1, \mathsf{false})$ and $P(t_3, a)$, then $P(\mathsf{if}\ t_1\ \mathsf{then}\ t_2\ \mathsf{else}\ t_3, a)$;

(IfNum) for all terms t_1, t_2 and t_3 and numeric values nv, if $t_1 \Rightarrow nv$ and (†) $P(t_1, nv)$, then $P(\mathsf{if}\ t_1\ \mathsf{then}\ t_2\ \mathsf{else}\ t_3, \mathsf{error})$;

(IfError) for all terms t_1, t_2 and t_3, if $t_1 \Rightarrow \mathsf{error}$ and (†) $P(t_1, \mathsf{error})$, $P(\mathsf{if}\ t_1\ \mathsf{then}\ t_2\ \mathsf{else}\ t_3, \mathsf{error})$;

(SuccBool) for all terms t and boolean values bv, if $t \Rightarrow bv$ and (†) $P(t, bv)$, then $P(\mathsf{succ}\ t, \mathsf{error})$;

(SuccNum) for all terms t and numeric values nv, if $t \Rightarrow nv$ and (†) $P(t, nv)$, then $P(\mathsf{succ}\ t, \mathsf{succ}\ nv)$;

(SuccError) for all terms t, if $t \Rightarrow \mathsf{error}$ and (†) $P(t, \mathsf{error})$, then $P(\mathsf{succ}\ t, \mathsf{error})$;

(PredBool) for all terms t, if $t \Rightarrow bv$ and (†) $P(t, bv)$, then $P(\mathsf{pred}\ t, \mathsf{error})$;

(PredZero) for all terms t, if $t \Rightarrow \mathsf{0}$ and (†) $P(t, \mathsf{0})$, then $P(\mathsf{pred}\ t, \mathsf{error})$;

(PredSucc) for all terms t and numeric values nv, if $t \Rightarrow \mathsf{succ}\ nv$ and (†) $P(t, \mathsf{succ}\ nv)$, then $P(\mathsf{pred}\ t, nv)$;

(PredError) for all terms t, if $t \Rightarrow \mathsf{error}$ and (†) $P(t, \mathsf{error})$, then $P(\mathsf{pred}\ t, \mathsf{error})$;

(IszeroBool) for all terms t and boolean values bv, if $t \Rightarrow bv$ and (†) $P(t, bv)$, then $P(\mathsf{iszero}\ t, \mathsf{error})$;

(IszeroZero) for all terms t, if $t \Rightarrow \mathsf{0}$ and (†) $P(t, \mathsf{0})$, then $P(\mathsf{iszero}\ t, \mathsf{true})$;

(IszeroSucc) for all terms t and numeric values nv, if $t \Rightarrow \mathsf{succ}\ nv$ and $P(t, \mathsf{succ}\ nv)$, then $P(\mathsf{iszero}\ t, \mathsf{false})$;

(IszeroError) for all terms t, if $t \Rightarrow \mathsf{error}$ and (†) $P(t, \mathsf{error})$, then $P(\mathsf{iszero}\ t, \mathsf{error})$;

(OtherwiseValue) for all terms t_1 and t_2 and values v, if $t_1 \Rightarrow v$ and (†) $P(t_1, v)$, then $P(t_1\ \mathsf{otherwise}\ t_2, v)$;

(OtherwiseError) for all terms t_1 and t_2 and answers a, if $t_1 \Rightarrow \mathsf{error}$ and $t_2 \Rightarrow a$, and (†) $P(t_1, \mathsf{error})$ and $P(t_2, a)$, then $P(t_1 \ \mathsf{otherwise} \ t_2, a)$.

We refer to (†) as the *inductive hypothesis*.

Exercise 1 (25 Points)

Prove that the evaluation relation ($\rightarrow$) is *deterministic*: for all terms t_1, t_2 and t_2', if $t_1 \rightarrow t_2$ and $t_1 \rightarrow t_2'$, then $t_2 = t_2'$.

Exercise 2 (25 Points)

Prove that the complete evaluation relation ($\Rightarrow$) is *deterministic*: for all terms t, and answers a and a', if $t \Rightarrow a$ and $t \Rightarrow a'$, then $a = a'$.

Exercise 3 (50 Points)

Define a relation $\rightsquigarrow$ between terms and answers by: for all terms t and answers a, $t \rightsquigarrow a$ iff $t \rightarrow^* a$.

Prove that $\rightsquigarrow \, = \, \Rightarrow$.