Simple Extensions

We take the simply typed lambda calculus—plus booleans and natural numbers—as a base, and consider a number of additions.

The Unit Type

New syntactic forms

$t ::= \ldots$	terms:
unit	constant unit
$v ::= \ldots$	values:
unit	constant unit
$T ::= \ldots$	types:
Unit	unit type

New typing rules: $\boxed{\Gamma \vdash t : T}$

$$\Gamma \vdash \mathrm{unit} : \mathrm{Unit} \quad \text{(T-Unit)}$$

New derived forms:

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$$t_1; t_2 \stackrel{\mathrm{def}}{=}$$

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New derived forms:

$$t_1; t_2 \stackrel{\mathrm{def}}{=} (\lambda x : \mathrm{Unit}.\, t_2)\, t_1, \quad \text{where}$$

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New typing rules: $\boxed{\Gamma \vdash t : T}$

$$\Gamma \vdash \mathrm{unit} : \mathrm{Unit} \quad \text{(T-Unit)}$$

New derived forms:

$$t_1; t_2 \stackrel{\mathrm{def}}{=} (\lambda x : \mathrm{Unit}.\, t_2)\, t_1, \quad \text{where } x \notin FV(t_2).$$

Ascription

New syntactic forms

$t ::= \ldots$	terms:
$t \text{ as } T$	ascription

New evaluation rules : $\boxed{t \longrightarrow t'}$

New typing rules: $\boxed{\Gamma \vdash t : T}$

Ascription

New syntactic forms

$$t ::= \ldots \qquad \text{terms:}$$

$$t \text{ as } T \qquad \text{ascription}$$

New evaluation rules : $\boxed{t \to t'}$

$$v_1 \text{ as } T \to v_1 \qquad \text{(E-Ascribe)}$$

$$\frac{t_1 \to t_1'}{t_1 \text{ as } T \to t_1' \text{ as } T} \qquad \text{(E-Ascribe1)}$$

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New typing rules: $\boxed{\Gamma \vdash t : T}$

$$\frac{\Gamma \vdash t_1 : T}{\Gamma \vdash t_1 \text{ as } T : T} \qquad \text{(T-Ascribe)}$$

Let Bindings

New syntactic forms

$t ::= \ldots$	terms:
$\text{let } x = t \text{ in } t$	let binding

New evaluation rules : $\boxed{t \longrightarrow t'}$

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Let Bindings

New syntactic forms

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New evaluation rules : $\boxed{t \longrightarrow t'}$

$$\text{let } x = v_1 \text{ in } t_2 \longrightarrow [x \mapsto v_1]t_2 \qquad \text{(E-LetV)}$$

$$\frac{t_1 \longrightarrow t_1'}{\text{let } x = t_1 \text{ in } t_2 \longrightarrow \text{let } x = t_1' \text{ in } t_2} \qquad \text{(E-Let)}$$

New typing rules: $\boxed{\Gamma \vdash t : T}$

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New typing rules: $\boxed{\Gamma \vdash t : T}$

$$\frac{\Gamma \vdash t_1 :}{\Gamma \vdash \text{let } x = t_1 \text{ in } t_2 : T_2} \qquad \text{(T-Let)}$$

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$$\text{let } x = v_1 \text{ in } t_2 \to [x \mapsto v_1]t_2 \qquad \text{(E-LetV)}$$

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New typing rules: $\boxed{\Gamma \vdash t : T}$

$$\frac{\Gamma \vdash t_1 : T_1 \qquad \vdash t_2 : T_2}{\Gamma \vdash \text{let } x = t_1 \text{ in } t_2 : T_2} \qquad \text{(T-Let)}$$

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New evaluation rules : $\boxed{t \longrightarrow t'}$

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$$\frac{t_1 \longrightarrow t_1'}{\text{let } x = t_1 \text{ in } t_2 \longrightarrow \text{let } x = t_1' \text{ in } t_2} \qquad \text{(E-Let)}$$

New typing rules: $\boxed{\Gamma \vdash t : T}$

$$\frac{\Gamma \vdash t_1 : T_1 \qquad \Gamma, x : T_1 \vdash t_2 : T_2}{\Gamma \vdash \text{let } x = t_1 \text{ in } t_2 : T_2} \qquad \text{(T-Let)}$$

Pairs

New syntactic forms

$t ::= \ldots$	terms:
$\{t, t\}$	pair
$t.1$	first projection
$t.2$	second projection
$v ::= \ldots$	values:
$\{v, v\}$	pair value
$T ::= \ldots$	types:
$T \times T$	product type

Pairs (Cont.)

New evaluation rules : $\boxed{t \to t'}$

$$\frac{t_1 \to t_1'}{\{t_1, t_2\} \to \{t_1', t_2\}} \qquad \text{(E-Pair1)}$$

$$\frac{t_2 \to t_2'}{\{\quad, t_2\} \to \{\quad, t_2'\}} \qquad \text{(E-Pair2)}$$

$$\frac{t_1 \to t_1'}{t_1.1 \to t_1'.1} \qquad \text{(E-Proj1)}$$

$$\frac{t_1 \to t_1'}{t_1.2 \to t_1'.2} \qquad \text{(E-Proj2)}$$

$$\{v_1, v_2\}.1 \to v_1 \qquad \text{(E-PairBeta1)}$$

$$\{v_1, v_2\}.2 \to v_2 \qquad \text{(E-PairBeta2)}$$

Pairs (Cont.)

New evaluation rules : $\boxed{t \to t'}$

$$\frac{t_1 \to t_1'}{\{t_1, t_2\} \to \{t_1', t_2\}} \qquad \text{(E-Pair1)}$$

$$\frac{t_2 \to t_2'}{\{v_1, t_2\} \to \{v_1, t_2'\}} \qquad \text{(E-Pair2)}$$

$$\frac{t_1 \to t_1'}{t_1.1 \to t_1'.1} \qquad \text{(E-Proj1)}$$

$$\frac{t_1 \to t_1'}{t_1.2 \to t_1'.2} \qquad \text{(E-Proj2)}$$

$$\{v_1, v_2\}.1 \to v_1 \qquad \text{(E-PairBeta1)}$$

$$\{v_1, v_2\}.2 \to v_2 \qquad \text{(E-PairBeta2)}$$

Pairs (Cont.)

New typing rules: $\boxed{\Gamma \vdash t : T}$

$$\frac{\Gamma \vdash t_1 : T_1 \qquad \Gamma \vdash t_2 : T_2}{\Gamma \vdash \{t_1, t_2\} : T_1 \times T_2} \quad \text{(T-Pair)}$$

$$\frac{\Gamma \vdash t_1 : T_{11} \times T_{12}}{\Gamma \vdash t_1.1 : T_{11}} \quad \text{(T-Proj1)}$$

$$\frac{\Gamma \vdash t_1 : T_{11} \times T_{12}}{\Gamma \vdash t_1.2 : T_{12}} \quad \text{(T-Proj2)}$$

Records

New syntactic forms

$t ::= \ldots$	terms:
$\{l_1 = t_1, \ldots, l_n = t_n\}$	record
$t.l$	projection
$v ::= \ldots$	values:
$\{l_1 = v_1, \ldots, l_n = v_n\}$	record value
$T ::= \ldots$	types:
$\{l_1 : T_1, \ldots, l_n : T_n\}$	type of records

Records (Cont.)

New evaluation rules : $\boxed{t \to t'}$

$$\{l_1 = v_1, \ldots, l_n = v_n\}.l_j \to v_j \qquad \text{(E-ProjRcd)}$$

$$\frac{t_1 \to t_1'}{t_1.l \to t_1'.l} \qquad \text{(E-Proj)}$$

$$\frac{t_j \to t_j'}{\begin{array}{l}\{l_1 = v_1, \ldots, l_{j-1} = v_{j-1}, l_j = t_j, \\ \quad l_{j+1} = t_{j+1}, \ldots, l_n = t_n\} \to \\ \{l_1 = v_1, \ldots, l_{j-1} = v_{j-1}, l_j = t_j', \\ \quad l_{j+1} = t_{j+1}, \ldots, l_n = t_n\}\end{array}} \qquad \text{(E-Rcd)}$$

Records (Cont.)

New typing rules: $\boxed{\Gamma \vdash t : T}$

$$\frac{\Gamma \vdash t_1 : T_1 \qquad \cdots \qquad \Gamma \vdash t_n : T_n}{\Gamma \vdash \{l_1 : t_1, \ldots, l_n : t_n\} : \{l_1 : T_1, \ldots, l_n : T_n\}} \quad \text{(T-Rcd)}$$

$$\frac{\Gamma \vdash t_1 : \{l_1 : T_1, \ldots, l_n : T_n\}}{\Gamma \vdash t_1.l_j : T_j} \quad \text{(T-Proj)}$$

Variants

New syntactic forms

$t ::= \ldots$	terms:
$\quad \langle l = t \rangle$ as T	tagging
$\quad$ case t of $\langle l_1 = x_1 \rangle \Rightarrow t_1 \mid \cdots \mid \langle l_n = x_n \rangle \Rightarrow t_n$	case
$v ::= \ldots$	values:
$\quad \langle l = v \rangle$ as T	tagging value
$T ::= \ldots$	types:
$\quad \langle l_1 : T_1, \ldots, l_n : T_n \rangle$	type of variants

Variants (Cont.)

New evaluation rules : $\boxed{t \to t'}$

$$\text{case } \langle l_j = v_j \rangle \text{ as } T \text{ of } \langle l_1 = x_1 \rangle \Rightarrow t_1 \mid \cdots \mid \langle l_n = x_n \rangle \Rightarrow t_n \to [x_j \mapsto v_j]t_j \qquad \text{(E-CaseVariant)}$$

$$\frac{t_0 \to t_0'}{\text{case } t_0 \text{ of } \langle l_1 = x_1 \rangle \Rightarrow t_1 \mid \cdots \mid \langle l_n = x_n \rangle \Rightarrow t_n \to \text{case } t_0' \text{ of } \langle l_1 = x_1 \rangle \Rightarrow t_1 \mid \cdots \mid \langle l_n = x_n \rangle \Rightarrow t_n} \qquad \text{(E-Case)}$$

$$\frac{t \to t'}{\langle l = t \rangle \text{ as } T \to \langle l = t' \rangle \text{ as } T} \qquad \text{(E-Variant)}$$

Variants (Cont.)

New typing rules: $\boxed{\Gamma \vdash t : T}$

$$\frac{\Gamma \vdash t_j : T_j}{\Gamma \vdash \langle l_j = t_j \rangle \text{ as } \langle l_1 : T_1, \ldots, l_n : T_n \rangle : \langle l_1 : T_1, \ldots, l_n : T_n \rangle} \quad \text{(T-Variant)}$$

$$\frac{\begin{array}{c}\Gamma \vdash t_0 : \langle l_1 : T_1, \ldots, l_n : T_n \rangle \\ \Gamma, x_1 : T_1 \vdash t_1 : T \quad \cdots \quad \Gamma, x_n : T_n \vdash t_n : T\end{array}}{\Gamma \vdash \text{case } t_0 \text{ of } \langle l_1 = x_1 \rangle \Rightarrow t_1 \mid \cdots \mid \langle l_n = x_n \rangle \Rightarrow t_n : T} \quad \text{(T-Case)}$$

General Recursion

New syntactic forms

$$t ::= \ldots \qquad \text{terms:}$$

$$\text{fix}\ t \qquad \text{fixed point of } t$$

New evaluation rules : $\boxed{t \longrightarrow t'}$

$$\text{fix}\ (\lambda x : T_1.t_2) \longrightarrow [x \mapsto \text{fix}\ (\lambda x : T_1.t_2)]t_2 \qquad \text{(E-FixBeta)}$$

$$\frac{t_1 \longrightarrow t_1'}{\text{fix}\ t_1 \longrightarrow \text{fix}\ t_1'} \qquad \text{(E-Fix)}$$

New typing rules: $\boxed{\Gamma \vdash t : T}$

$$\frac{}{\text{fix}\ t_1 : T_1} \qquad \text{(T-Fix)}$$

General Recursion

New syntactic forms

$t ::= \ldots$	terms:
$\mathsf{fix}\, t$	fixed point of t

New evaluation rules : $\boxed{t \longrightarrow t'}$

$$\mathsf{fix}\,(\lambda x : T_1.t_2) \longrightarrow [x \mapsto \mathsf{fix}\,(\lambda x : T_1.t_2)]t_2 \quad \text{(E-FixBeta)}$$

$$\frac{t_1 \longrightarrow t_1'}{\mathsf{fix}\, t_1 \longrightarrow \mathsf{fix}\, t_1'} \quad \text{(E-Fix)}$$

New typing rules: $\boxed{\Gamma \vdash t : T}$

$$\frac{\Gamma \vdash t_1 : T_1 \longrightarrow T_1}{\mathsf{fix}\, t_1 : T_1} \quad \text{(T-Fix)}$$