

Untyped Lambda Calculus: Syntax

$t ::=$	terms:
x	variables
$\lambda x. t$	abstraction
$t t$	application

$v ::=$	values:
$\lambda x. t$	abstraction value

Application associates to the left.

Abstractions extend as far to the right as possible.

$\lambda x. t$ binds x in t . Terms that differ only in the names of bound variables are interchangeable in all contexts.

1

Untyped Lambda Calculus: Free Variables and Substitution

The *free variables* of a term are defined recursively as follows:

$$\begin{aligned} \text{FV}(x) &= \{x\}, \\ \text{FV}(\lambda x. t_1) &= \text{FV}(t_1) \setminus \{x\}, \\ \text{FV}(t_1 t_2) &= \text{FV}(t_1) \cup \text{FV}(t_2). \end{aligned}$$

A term is *closed* iff it has no free variables; otherwise it is *open*.

The *substitution* of s for the free occurrences of x in t is defined recursively by:

$$\begin{aligned} [x \mapsto s]x &= s, \\ [x \mapsto s]y &= y, \text{ if } y \neq x, \\ [x \mapsto s](\lambda y. t_1) &= \lambda y. [x \mapsto s]t_1, \text{ if } y \neq x \text{ and } y \notin \text{FV}(s), \\ [x \mapsto s](t_1 t_2) &= ([x \mapsto s]t_1 [x \mapsto s]t_2). \end{aligned}$$

2

Untyped Lambda Calculus: Evaluation

Evaluation: $t \rightarrow t'$

$$\frac{t_1 \rightarrow t'_1}{t_1 t_2 \rightarrow t'_1 t_2} \quad (\text{E-App1})$$

$$\frac{t_2 \rightarrow t'_2}{v_1 t_2 \rightarrow v_1 t'_2} \quad (\text{E-App2})$$

$$(\lambda x. t_{12})v_2 \rightarrow [x \mapsto v_2]t_{12} \quad (\text{E-AppAbs})$$

It doesn't make much sense to try to evaluate open terms. Thus the subtleties of substitution don't really come into play in this evaluation relation.