

Breadth Exam in Formal Language Theory

Fall 2007

Duration: 90 Minutes

Format: Closed-book, Closed-notes

Question 1 (25 points)

Find a grammar G such that $L(G) = \{0^i 1^j 2^k 3^l \mid i, j, k, l \in \mathbb{N} \text{ and } i + j = k + l\}$.

Question 2 (75 points)

Given a string $w \in \{0, 1\}^*$, we write $\mathbf{diff}(w)$ (“difference”) for

the number of 1’s in $w - 2$ (the number of 0’s in w).

Then:

- $\mathbf{diff}(\epsilon) = 0 - 2 \times 0 = 0 - 0 = 0$;
- $\mathbf{diff}(1) = 1 - 2 \times 0 = 1 - 0 = 1$;
- $\mathbf{diff}(0) = 0 - 2 \times 1 = 0 - 2 = -2$;
- for all $x, y \in \{0, 1\}^*$, $\mathbf{diff}(xy) = \mathbf{diff}(x) + \mathbf{diff}(y)$.

For example, $\mathbf{diff}(10010) = 2 - 2 \times 3 = 2 - 6 = -4$.

Let

$$X = \{w \in \{0, 1\}^* \mid \text{for all prefixes } v \text{ of } w, 0 \leq \mathbf{diff}(v) \leq 3\}.$$

(a) Find a DFA M such that $L(M) = X$. [25 points]

(b) Prove that your answer to Part (a) is correct. [50 points]