Breadth Exam in Formal Language Theory
Fall 2009

Duration: 90 Minutes
Format: Closed-book, Closed-notes, Closed-computers

Note that elegance and simplicity of solutions will be taken into account when grading.

Question 1 (25 points)

Find a context-free grammar G such that $L(G) = \{ \mathsf{0}^i\mathsf{1}^j\mathsf{2}^k \mid i, j, k \in \mathbb{N} \text{ and either } i = j + k \text{ or } i + j = k \}$.

Question 2 (75 points)

Let f be the unique function from $\{\mathsf{0},\mathsf{1}\}^*$ to $\mathbb{N}$ such that:

- $f(\epsilon) = 0$;
- for all $w \in \{\mathsf{0},\mathsf{1}\}^*$,

$$f(w\mathsf{0}) = \begin{cases} f(w) + 1, & \text{if } \mathsf{01} \text{ is a suffix of } w, \\ f(w), & \text{if } \mathsf{01} \text{ is not a suffix of } w; \end{cases}$$

- for all $w \in \{\mathsf{0},\mathsf{1}\}^*$, $f(w\mathsf{1}) = f(w)$.

Thus, for all $w \in \{\mathsf{0},\mathsf{1}\}^*$, $f(w)$ is the number of possibly overlapping occurrences of $\mathsf{010}$ in w. E.g., $f(\mathsf{01010}) = 2$. Let $X = \{ w \in \{\mathsf{0},\mathsf{1}\}^* \mid f(w) \text{ is odd} \}$.

(a) Find a DFA M such that $L(M) = X$. [25 points]

(b) Prove that your answer to Part (a) is correct. Make your proof as complete and rigorous as possible. [50 points]