Breadth Exam in Formal Language Theory
Spring 2007

Duration: 90 Minutes
Format: Closed-book, Closed-notes

Question 1 (25 points)

Find a grammar G such that $L(G) = \{ \mathsf{0}^i\mathsf{1}^j\mathsf{2}^k\mathsf{3}^l \mid i, j, k, l \in \mathbb{N} \text{ and } i + j + k = l \}$.

Question 2 (75 points)

Given a string $w \in \{\mathsf{0}, \mathsf{1}\}^*$, we write **diff**$(w)$ ("difference") for

$$\text{the number of } \mathsf{1}\text{'s in } w - \text{the number of } \mathsf{0}\text{'s in } w.$$

Then:

- **diff**$(\epsilon) = 0 - 0 = 0$;
- **diff**$(\mathsf{1}) = 1 - 0 = 1$;
- **diff**$(\mathsf{0}) = 0 - 1 = -1$;
- for all $x, y \in \{\mathsf{0}, \mathsf{1}\}^*$, **diff**$(xy) =$ **diff**$(x) +$ **diff**(y).

For example, **diff**$(\mathsf{10010}) = 2 - 3 = -1$.

Let

$$X = \{ w \in \{\mathsf{0}, \mathsf{1}\}^* \mid \textbf{diff}(w) = 3n \text{ for some } n \in \mathbb{Z} \}.$$

That is, X consists of all elements of $\{\mathsf{0}, \mathsf{1}\}^*$ whose **diff**'s are divisible by 3.

(a) Find a DFA M such that $L(M) = X$. [25 points]

(b) Prove that your answer to Part (a) is correct. [50 points]