

Breadth Exam in Formal Language Theory

Spring 2009

Duration: 90 Minutes

Format: Closed-book, Closed-notes, Closed-computers

Note that elegance and simplicity of solutions will be taken into account when grading.

Question 1 (25 points)

Find a context-free grammar G such that $L(G) = \{0^i1^j2^k \mid i + j = 2k\}$.

Question 2 (75 points)

Let $X = \{w \in \{0, 1, 2\}^* \mid \text{for all } a, b \in \{0, 1, 2\}, \text{ if } ab \text{ is a substring of } w, \text{ then } ab \in \{01, 12, 20\}\}$.

(a) Find a DFA M such that $L(M) = X$. [25 points]

(b) Prove that your answer to Part (a) is correct. Make your proof as complete and rigorous as possible. [50 points]