CS 591 S2—Formal Language Theory: Integrating Experimentation and Proof—Fall 2019

Final Examination

Monday, December 16, 12:30–2:30pm

Question 1 (20 points)

Suppose we know that $i, i', j, j', k, k' \in \mathbb{N}$ and

$$0^i 1^j 0^k = 0^{i'} 1^{j'} 0^{k'}.$$

What can we conclude about the relationship between i, j, k and i', j', k'? (You don't need to prove that your answer is correct.)

Question 2 (20 points)

Let $X = \{\, 0^i 1^j 2^k \mid i, j, k \in \mathbb{N} \text{ and } 0 < i + j < k \,\}$. Find a grammar G such that $L(G) = X$.

Question 3 (20 points)

Given $w \in \{0, 1\}^*$, we write:

- **zeros** w for the number of occurrences of 0 in w; and
- **ones** w for the number of occurrences of 1 in w.

Define the following languages:

$$\begin{aligned}
X_{\mathsf{ee}} &= \{\, w \in \{0,1\}^* \mid \textbf{zeros}\, w \text{ is even and } \textbf{ones}\, w \text{ is even} \,\},\\
X_{\mathsf{eo}} &= \{\, w \in \{0,1\}^* \mid \textbf{zeros}\, w \text{ is even and } \textbf{ones}\, w \text{ is odd} \,\},\\
X_{\mathsf{oe}} &= \{\, w \in \{0,1\}^* \mid \textbf{zeros}\, w \text{ is odd and } \textbf{ones}\, w \text{ is even} \,\},\\
X_{\mathsf{oo}} &= \{\, w \in \{0,1\}^* \mid \textbf{zeros}\, w \text{ is odd and } \textbf{ones}\, w \text{ is odd} \,\}.
\end{aligned}$$

Assume that we have already proven the following facts: (1) $\% \in X_{\mathsf{ee}}$; (2) $X_{\mathsf{ee}}\{0\} \subseteq X_{\mathsf{oe}}$; (3) $X_{\mathsf{ee}}\{1\} \subseteq X_{\mathsf{eo}}$; (4) $X_{\mathsf{eo}}\{0\} \subseteq X_{\mathsf{oo}}$; (5) $X_{\mathsf{eo}}\{1\} \subseteq X_{\mathsf{ee}}$; (6) $X_{\mathsf{oe}}\{0\} \subseteq X_{\mathsf{ee}}$; (7) $X_{\mathsf{oe}}\{1\} \subseteq X_{\mathsf{oo}}$; (8) $X_{\mathsf{oo}}\{0\} \subseteq X_{\mathsf{eo}}$; and (9) $X_{\mathsf{oo}}\{1\} \subseteq X_{\mathsf{oe}}$.

Find a DFA M such that $L(M) = X_{\mathsf{eo}} \cup X_{\mathsf{oe}}$, and prove that your answer is correct.

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Question 4 (20 points)

Let the language X be

$$\{\, \mathsf{0}^i\mathsf{1}^j\mathsf{2}^k\mathsf{3}^l \mid i, j, k, l \in \mathbb{N} \text{ and } i + j = k + l \text{ and } i \text{ is even and } j \text{ is odd and } k \text{ is even and } l \text{ is odd} \,\}.$$

Prove that X is not regular.

Question 5 (20 points)

Suppose α and β are regular expressions whose alphabets are subsets of $\{\mathsf{0}, \mathsf{1}\}^*$. Let

$$X = \{\, w \in \{\mathsf{0}, \mathsf{1}\}^* \mid \text{for all } x, y \in \{\mathsf{0}, \mathsf{1}\}^*, \text{ if } w = xy, \text{ then, if } x \in L(\alpha), \text{ then } y \notin L(\beta) \,\}.$$

Explain how we can use the algorithms we've studied to create a regular expression γ such that $L(\gamma) = X$. (You don't need to worry about making γ small.)