CS 591 S2—Formal Language Theory: Integrating Experimentation and Proof—Fall 2019

Problem Set 1

Due by 12:30pm on Thursday, September 19

Problem 1 (25 points)

(a) Either prove or disprove the following statement:

For all sets A, B and C,

$$A - (B \cup C) = (A - B) - C.$$

[12 points]

(b) Either prove or disprove the following statement:

For all sets A, B and C,

$$A - (B \cap C) = (A - B) \cup (A - C).$$

[13 points]

Problem 2 (25 points)

Prove that, for all $n \in \mathbb{N}$, if $n \geq 18$, then there are $i, j \in \mathbb{N}$ such that $n = 4i + 7j$.

Problem 3 (25 points)

Prove that $\mathbb{N} \cong \mathbb{N} \times \mathbb{N}$. Hint, consider using the approach of Exercise 1.1.8, which uses the Schröder-Bernstein Theorem, as well as the following consequence of the Fundamental Theorem of Arithmetic: if two ascending (each element is $\leq$ the next) lists of prime numbers (natural numbers that are at least 2 and have no divisors other than 1 and themselves) have the same product (the product of the empty list is 1), then they are equal.

Problem 4 (25 points)

Suppose A is a set, R is a relation on A, and R is *not* well-founded on A. Suppose we were allowed to use Theorem 1.2.8 (Principle of Well-founded Induction) with R (even though it is not well-founded). Prove an obviously false statement; try to make your statement as simple as possible.