CS 591 S2—Formal Language Theory: Integrating Experimentation and Proof—Fall 2019

Problem Set 6

Due by 12:30pm on Tuesday, November 26

Problem 1 (25 points)

Define a function $\mathbf{diff} \in \{0,1\}^* \to \mathbb{Z}$ by: for all $w \in \{0,1\}^*$,

$$\mathbf{diff}\, w = \text{the number of 1's in } w - 2(\text{the number of 0's in } w).$$

Thus

- $\mathbf{diff}\, \% = 0$;
- $\mathbf{diff}\, 0 = -2$;
- $\mathbf{diff}\, 1 = 1$;
- for all $x, y \in \{0,1\}^*$, $\mathbf{diff}(xy) = \mathbf{diff}\, x + \mathbf{diff}\, y$.

Let $Y = \{\, w \in \{0,1\}^* \mid \text{for all prefixes } v \text{ of } w, \mathbf{diff}\, v \geq 0 \,\}$. Prove that Y is not regular.

Problem 2 (50 points)

Consider the language Y of Problem 1. Find a grammar G such that $L(G) = Y$, and prove that your answer is correct. (Hint: you may make use of whatever results we have already proved about this language.)

Problem 3 (25 points)

Let G be the grammar

$$\begin{aligned}
\mathsf{A} &\to \mathsf{BE} \mid \mathsf{DC} \\
\mathsf{B} &\to 1 \mid \mathsf{B}1 \mid 0\mathsf{B}1 \\
\mathsf{C} &\to 2 \mid \mathsf{C}2 \mid 1\mathsf{C}2 \\
\mathsf{D} &\to \% \mid 0\mathsf{D} \\
\mathsf{E} &\to \% \mid 2\mathsf{E}
\end{aligned}$$

(a) Describe $L(G)$ as simply as possible, without mentioning G. [12 points]

(b) Find and draw a pair of parse trees showing that G is ambiguous. Use Forlan to show that your answer is correct. (Include a transcript of your Forlan session.) [13 points]