CS 516—Software Foundations via Formal Languages—Spring 2025

Problem Set 6

Due by 11:59pm on Thursday, April 17

Problem 1 (20 points)

Define a function **diff** $\in \{0,1\}^* \to \mathbb{Z}$ by: for all $w \in \{0,1\}^*$,

$$\textbf{diff}\, w = \text{the number of 1's in } w - 2(\text{the number of 0's in } w).$$

Thus

- **diff** $\% = 0$;
- **diff** $0 = -2$;
- **diff** $1 = 1$;
- for all $x, y \in \{0,1\}^*$, **diff**$(xy) =$ **diff** $x +$ **diff** y.

Let $Y = \{\, w \in \{0,1\}^* \mid \text{for all prefixes } v \text{ of } w, \textbf{diff}\, v \geq 0 \,\}$. Prove that Y is not regular.

Problem 2 (20 points)

Let $L = \{\, 0^i 1^j 2^k 3^l \mid i, j, k, l \in \mathbb{N} \text{ and either: } i < k \text{ or } j > l \,\}$. Find a grammar G such that $L(G) = L$. (You can test your definition of G (see Section 4.3 of the slides/book), but this is not required.)

Problem 3 (60 points)

Define a function **DCS** (for "disjoint, correlated substrings") from $\{0,1\}^* \times \{0,1\}^*$ to $\mathcal{P}(\{0,1\}^*)$ by: for all $x, y \in \{0,1\}^*$, **DCS**(x, y) is the set of all $w \in \{0,1\}^*$ such that:

- for all $u, v \in \{0,1\}^*$, if $w = uxv$, then y is a substring of u or v; and
- for all $u, v \in \{0,1\}^*$, if $w = uyv$, then x is a substring of u or v.

(a) Use the functions/algorithms given in the slides/book to define a function/algorithm **dcsDFA** $\in \{0,1\}^* \times \{0,1\}^* \to$ **DFA** such that, for all $x, y \in \{0,1\}^*$:

- $L($**dcsDFA**$(x, y)) =$ **DCS**(x, y); and

- **minimize**(**dcsDFA**(x, y)) is isomorphic to **dcsDFA**(x, y).

[20 points]

(b) In the file `ps6.sml`, define a Forlan/SML function

```
val dcsDFA : str * str -> dfa
```

that implements your definition of **dcsDFA**. You should assume that `dcsDFA` will only be called with elements of $\{0, 1\}^*$. Load your file into Forlan, and execute the following:

```
val dfa1 = dcsDFA(Str.fromString "11", Str.fromString "00");
DFA.numStates dfa1;
val dfa2 = dcsDFA(Str.fromString "011", Str.fromString "110");
DFA.numStates dfa2;
```

(Include a transcript of your Forlan session.) Put `ps6.sml` in the subdirectory `CS516-PS6` of your private GitHub repository. [20 points]

(c) Prove that your definition of **dcsDFA** is correct. Obvious steps like $L(\textbf{minimize}\, M) = L(M)$ don't have to be written out; instead you can refer to "simple calculations". [20 points]